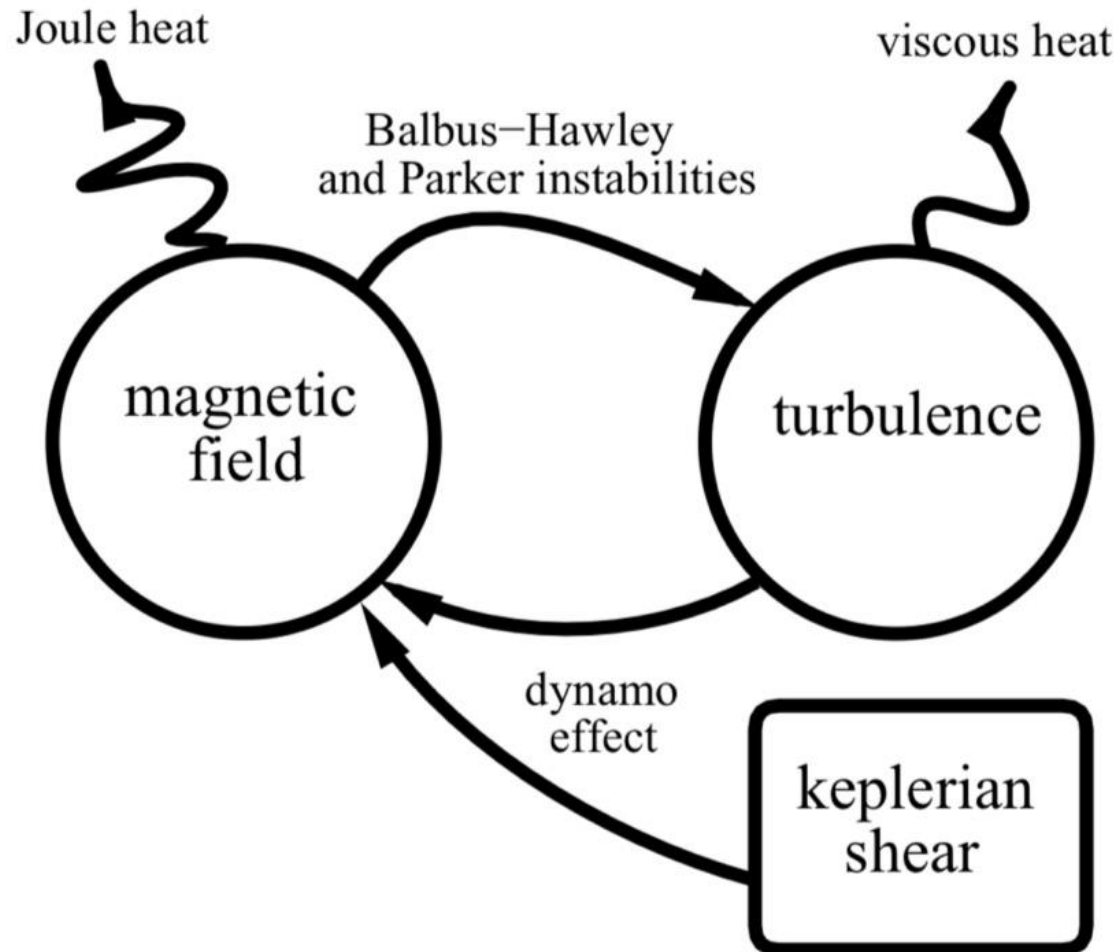


Interplays between dynamo and magneto-rotational instabilities

Axel Brandenburg (Nordita, Stockholm)



- Original motivation for MRI
 - Turbulence in accretion disks
 - Zeldovich in Potsdam?
- But need a magnetic field
 - Turbulence \rightarrow dynamo
 - Reinforce MRI
- Dynamo
 - Still struggling with problems
 - Dependence on microphysical magnetic diffusivity
 - Insights from cosmology
- Dissipation
 - Depends on magnetic Prandtl number

Recent motivation for MRI in the Sun

Article

The solar dynamo begins near the surface

<https://doi.org/10.1038/s41586-024-07315-1>

Received: 19 August 2023

Accepted: 14 March 2024

Published online: 22 May 2024

Open access

 Check for updates

Geoffrey M. Vasil^{1✉}, Daniel Lecoanet^{2,3}, Kyle Augustson^{2,3}, Keaton J. Burns^{4,5}, Jeffrey S. Oishi⁶, Benjamin P. Brown⁷, Nicholas Brummell⁸ & Keith Julien^{9,10}

The magnetic dynamo cycle of the Sun features a distinct pattern: a propagating region of sunspot emergence appears around 30° latitude and vanishes near the equator every 11 years (ref. 1). Moreover, longitudinal flows called torsional oscillations closely shadow sunspot migration, undoubtedly sharing a common cause². Contrary

for an extremely under-constrained process. Our turbulent parameterizations also produce falsifiable predictions: our proposed MRI **dynamo** mechanism would face severe challenges if future helioseismic studies of the Sun suggest that the turbulent dissipation is much larger than expected (for example, if the effective $Re \ll 1$). However, it is difficult to imagine how any nonlinear dynamics would happen in this scenario.

- No turbulence
 - Solar dynamo works in spite of turbulence?
- Shear flow given
 - In reality by Λ effect
- Magnetic field given
 - In reality by α effect
- Replaces Ω effect?

Early work on MRI in the Sun

Mon. Not. R. Astron. Soc. **280**, 149–152 (1996)

On hydrodynamic stability of weakly magnetized stellar radiative zones

V. A. Urpin

A. F. Ioffe Institute for Physics and Technology, SU 194021, St Petersburg, Russia

Accepted 1995 November 14. Received 1995 November 13; in original form 1995 May 1

ABSTRACT

We consider the stability of weakly magnetized differentially rotating stars to adiabatic axisymmetric disturbances. The general case of the angular velocity being dependent on both the spherical radius and the polar angle is analysed. In magnetized radiative zones, there are two low-frequency modes, the buoyant and torsion Alfvén ones, which can be destabilized by a differential rotation. For instability, the buoyant mode requires a decrease of the specific angular momentum in the direction from the poles to the equator, whereas the Alfvén mode can be unstable if the angular velocity decreases from the poles to the equator. A relatively strong magnetic field may stabilize both of these modes.

Key words: instabilities – MHD – stars: interiors – stars: magnetic fields.

- Applied to radiative zones
 - Buoyancy instability
 - MRI
- Predates Spruit dynamo
 - Inspired Tayler-Spruit dynamo

Popular application to proto-neutronstars

Impact of magnetohydrodynamic turbulence on thermal wind balance in the Sun

Youhei Masada^{1,2★}

¹*Department of Computational Science, Kobe University, 657-8501 Kobe, Japan*

²*Hinode Science Center, National Astronomical Observatory of Japan, 181-8588 Mitaka, Japan*

Accepted 2010 November 9. Received 2010 November 2; in original form 2010 October 2

ABSTRACT

The possible role of magnetorotational instability (MRI) and its driven magnetohydrodynamic (MHD) turbulence in the solar interior is studied on the basis of linear and non-linear theories coupled with physical parameters, assuming a solar rotation profile inverted from helioseismic observations and a standard model for the internal structure of the Sun. We find that the location of MRI is confined to the higher latitude tachocline and lower latitude near-surface shear layer. It is especially interesting that the MRI-active region around the tachocline closely overlaps with the area indicating a steep entropy rise, which is required from the thermal wind balance in the Sun. This suggests that the MRI-driven turbulence plays a crucial role in maintaining the thermal wind balance in the Sun via the exceptional turbulent heating and equatorward angular momentum transport. The warm pole existing around the tachocline might be a natural outcome of the turbulent activities energized by the MRI.

- Competes with neutrino-driven convection
 - Convection near poles
 - MRI near equator
- Studied by many people
 - ...

Instability on top of convective instability?

THE DIFFERENTIAL ROTATION OF THE SOLAR SURFACE

PETER J. GIERASCH

Center for Radiophysics and Space Research, Cornell University, Ithaca, New York 14850

Received 1973 August 14; revised 1973 November 15

ABSTRACT

The large-scale flow in the solar convection zone is discussed. The objective is to deduce from observation the principal physical balances in the governing equations. The simplest set of equations that seem potentially realistic are utilized. It is assumed that magnetic fields are negligible, that **mixing-length theory** gives an accurate representation of the **mean structure**, and that rigid-body rotation exists at the base. It is deduced that the zonal flow is geostrophic, the meridional flow is controlled by friction, and diffusive heating balances advective cooling due to vertical motion. Next, a detailed calculation of the latitude profile of surface angular velocity is performed, based on approximate equations containing only the principal balances, and agrees well with observation. Finally, it is demonstrated that the amplitude of the flow may be determined by the constraint that the net equatorward angular momentum flux vanish. The conclusions are consistent with the hypothesis that the flow is primarily a single large axisymmetric convection cell in each hemisphere. There is contradiction with the observation of no equator-pole flux variation, but this may be due to oversimplification of equations and boundary conditions.

- Alternative to Λ effect?
 - Nature of instability
- CC
 - Turbulence \rightarrow dynamo
 - Reinforce MRI
- Dynamo
 - Still struggling with problems
 - Dependence on microphysical magnetic diffusivity
 - Insights from cosmology
- Dissipation
 - Depends on magnetic Prandtl number

Does solar differential rotation arise from a large scale instability?

Ilkka Tuominen¹, Axel Brandenburg^{2,3}, David Moss⁴, and Michel Rieutord⁵

¹ Observatory and Astrophysics Laboratory, P.O. Box 14, SF-00014 University of Helsinki,

² HAO/NCAR*, P.O. Box 3000, Boulder, CO 80307, USA

³ NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

⁴ Mathematics Department, The University, Manchester M13 9PL, UK

⁵ Observatoire Midi-Pyrénées, 14 av. E. Belin, F-31400 Toulouse, France

Received 6 May 1993 / Accepted 26 September 1993

is the mean-field energy equation

$$\langle \rho \rangle \langle T \rangle \left(\frac{\partial}{\partial t} + \langle \mathbf{u} \rangle \cdot \nabla \right) \langle s \rangle = \nabla \cdot (\chi_t \langle \rho \rangle \langle T \rangle \nabla \langle s \rangle), \quad (1)$$

(Durney & Roxburgh 1971), where χ_t is the turbulent heat conductivity. This equation is rather similar to the original energy equation

$$\rho T \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) s = \nabla \cdot (\chi \rho c_p \nabla T), \quad (2)$$

instability. The Rayleigh number now has to be defined using turbulent coefficients for viscosity and conductivity:

$$\text{Ra}_t = - \frac{gd^4}{\nu_t \chi_t c_p} \frac{1}{dr} \frac{ds_0}{dr}, \quad (5)$$

where g is gravity, d the thickness of the layer, and s_0 is the entropy gradient of the quasi-hydrostatic reference solution. It

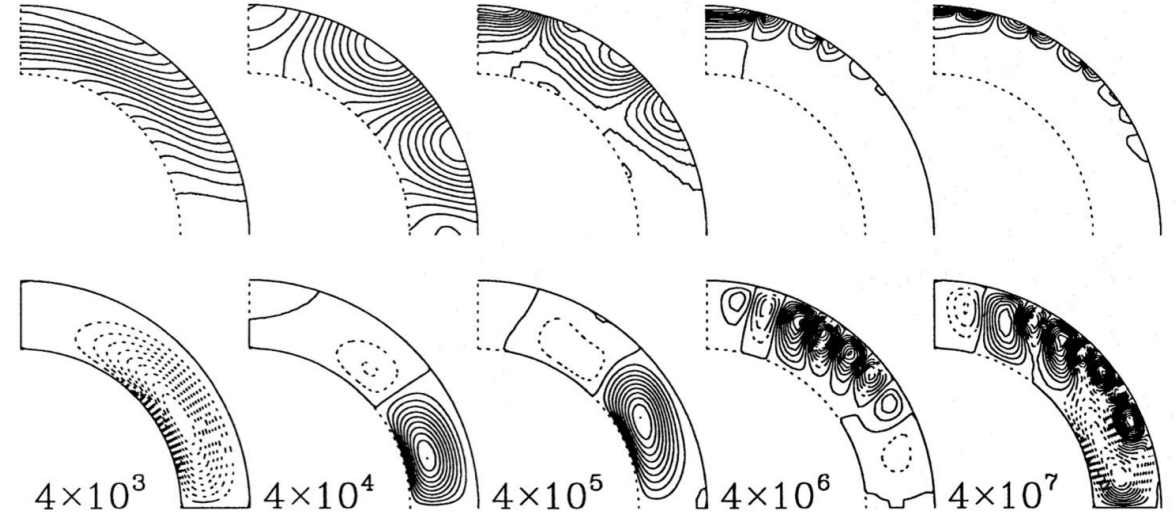


Table 2. Sketch of the bifurcation sequence for laboratory Rayleigh-Bénard convection. The critical values of Ra are taken from Heslot et al. (1987)

Ra	convection	large scale flow
$6 \cdot 10^3$	onset of convection	
$9 \cdot 10^4$	oscillatory convection	
$1.5 \cdot 10^5$	chaotic convection	
$3 \cdot 10^5$	soft turbulence	
$\approx 2 \cdot 10^6$		onset of large scale flow
$4 \cdot 10^7$	hard turbulence	
$\approx 10^{13}$		osc. large scale flow

Quantifying the effect of turbulent magnetic diffusion on the growth rate of the magneto–rotational instability[★]

M. S. Väisälä^{1,2}, A. Brandenburg^{2,3}, D. Mitra², P. J. Käpylä^{1,2,3}, and M. J. Mantere⁴

¹ Department of Physics, Gustaf Hällströmin katu 2a, PO Box 64, 00014 University of Helsinki, Finland
e-mail: miikka.vaisala@helsinki.fi

² Nordita, KTH Royal Institute of Technology and Stockholm University, Roslagstullsbacken 23, 10691 Stockholm, Sweden

³ Department of Astronomy, AlbaNova University Center, Stockholm University, 10691 Stockholm, Sweden

⁴ Aalto University, ReSoLVE Centre of Excellence, Department of Information and Computer Science, PO Box 15400, 00076 Aalto, Finland

Received 11 October 2013 / Accepted 9 May 2014

ABSTRACT

Context. In astrophysics, turbulent diffusion is often used in place of microphysical diffusion to avoid resolving the small scales. However, we expect this approach to break down when time and length scales of the turbulence become comparable with other relevant time and length scales in the system. Turbulent diffusion has previously been applied to the magneto-rotational instability (MRI), but no quantitative comparison of growth rates at different turbulent intensities has been performed.

Aims. We investigate to what extent turbulent diffusion can be used to model the effects of small-scale turbulence on the kinematic growth rates of the MRI, and how this depends on angular velocity and magnetic field strength.

Methods. We use direct numerical simulations in three-dimensional shearing boxes with periodic boundary conditions in the spanwise direction and additional random plane-wave volume forcing to drive a turbulent flow at a given length scale. We estimate the turbulent diffusivity using a mixing length formula and compare with results obtained with the test-field method.

Results. It turns out that the concept of turbulent diffusion is remarkably accurate in describing the effect of turbulence on the growth rate of the MRI. No noticeable breakdown of turbulent diffusion has been found, even when time and length scales of the turbulence become comparable with those imposed by the MRI itself. On the other hand, quenching of turbulent magnetic diffusivity by the magnetic field is found to be absent.

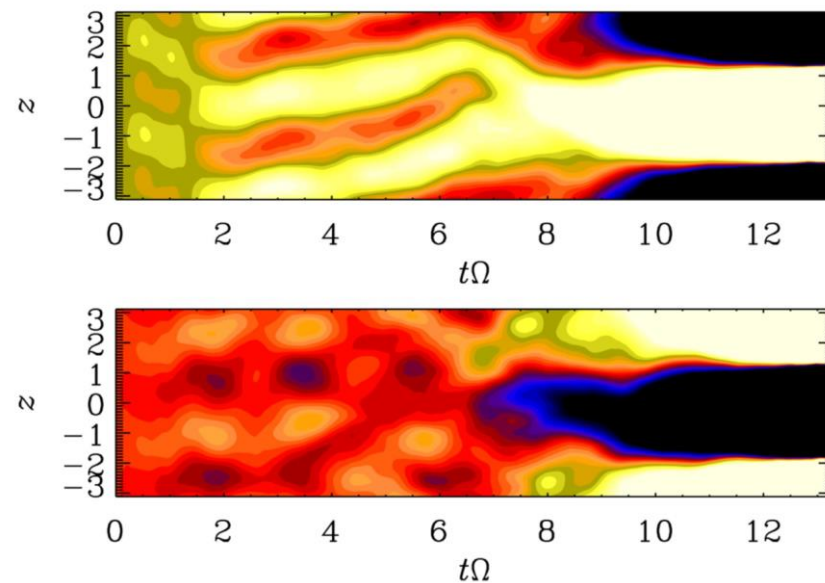
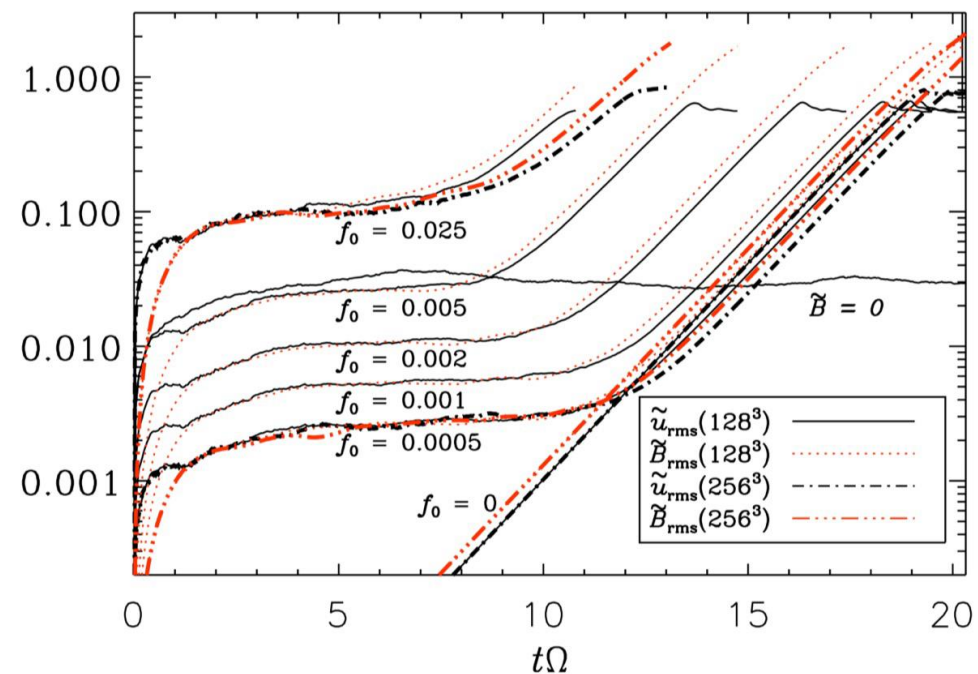
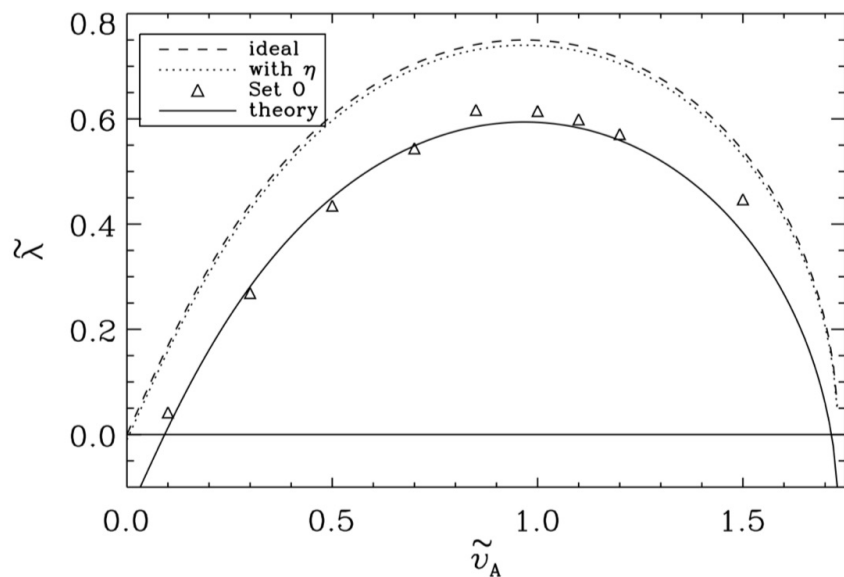
Conclusions. Turbulence reduces the growth rate of the MRI in the same way as microphysical magnetic diffusion does.

Growth rates well reproduced

$$\lambda \approx V_A(k)k - \eta_T k^2, \quad (1)$$

where $V_A(k)k$ is the growth rate in the non-turbulent, ideal case. For the MRI with Keplerian shear, $V_A(k)$ is given in terms of $\tilde{V}_A = V_A k / \Omega$ with (Balbus & Hawley 1998)

$$V_A(k)^2 = \left(\tilde{v}_A^2 + \frac{1}{2} \right) \left\{ \left[1 + 4 \frac{(3 - \tilde{v}_A^2) \tilde{v}_A^2}{(2\tilde{v}_A^2 + 1)^2} \right]^{1/2} - 1 \right\}, \quad (2)$$

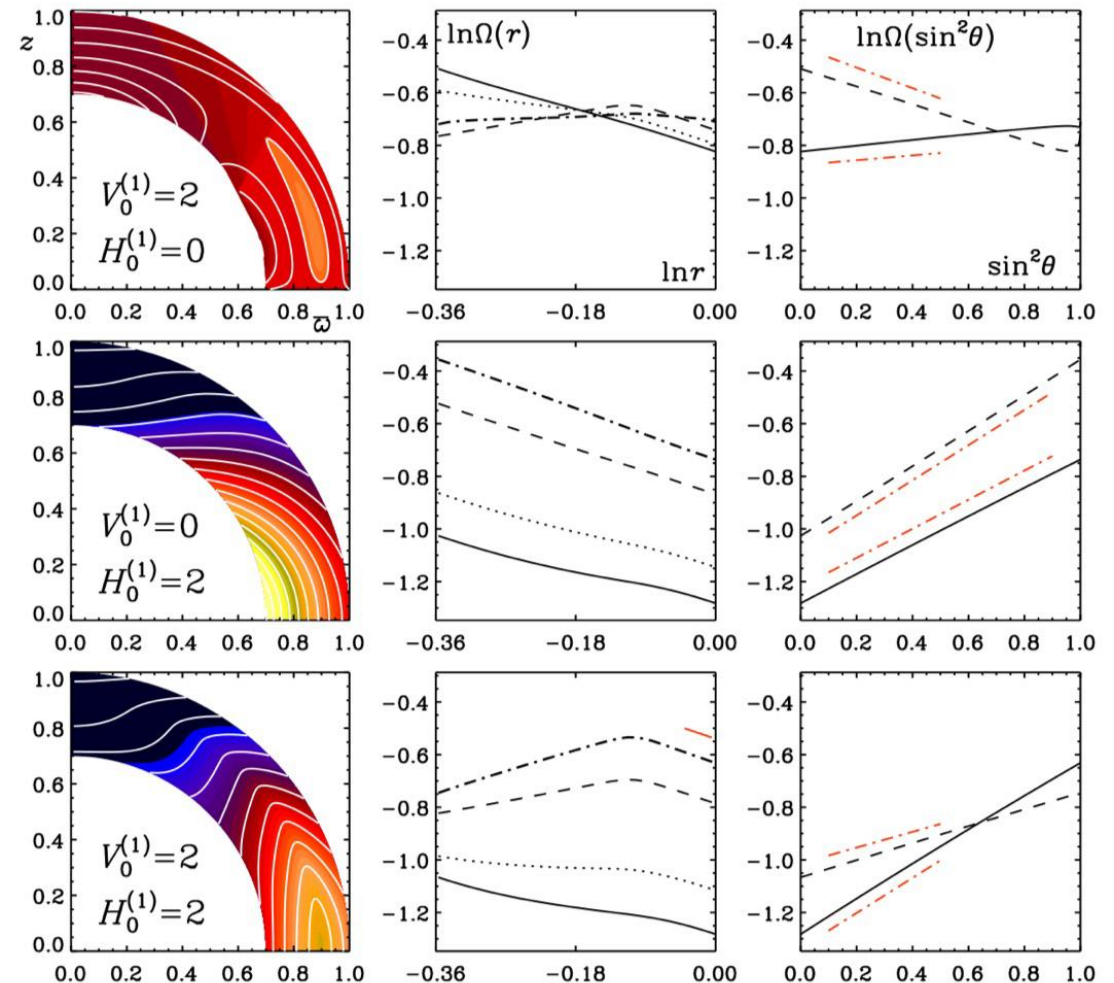


Magneto-rotational instability in a solar mean-field model

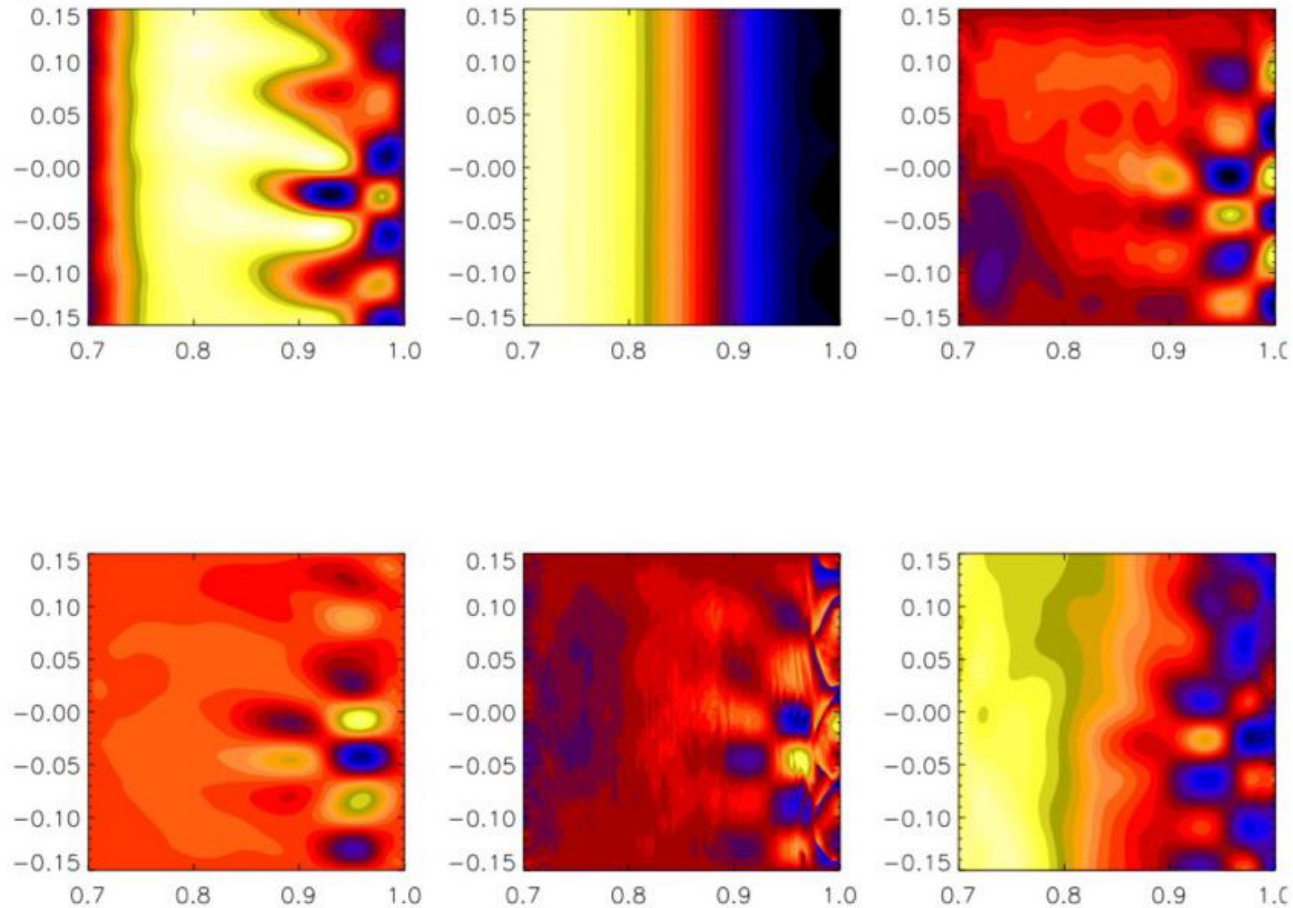
$$\bar{\rho} \frac{D\bar{U}}{Dt} = \bar{\mathbf{J}} \times \bar{\mathbf{B}} - \nabla \bar{p} - \nabla \cdot \bar{\mathbf{Q}},$$

$$\bar{Q}_{ij} = \Lambda_{ijk} \Omega_k - \nu_t \left(\bar{U}_{i;j} + \bar{U}_{j;i} - \frac{1}{3} \delta_{ij} \bar{U}_{k;k} \right),$$

- Near-surface shear layer rather thin
 - Difficult to get proper MRI
- At realistic turbulent Taylor numbers
 - Ω contour always become cylindrical
- Could be fixed by baroclinic term
 - Not pursued so far
- Return to distributed models with negative radial Ω gradient throughout

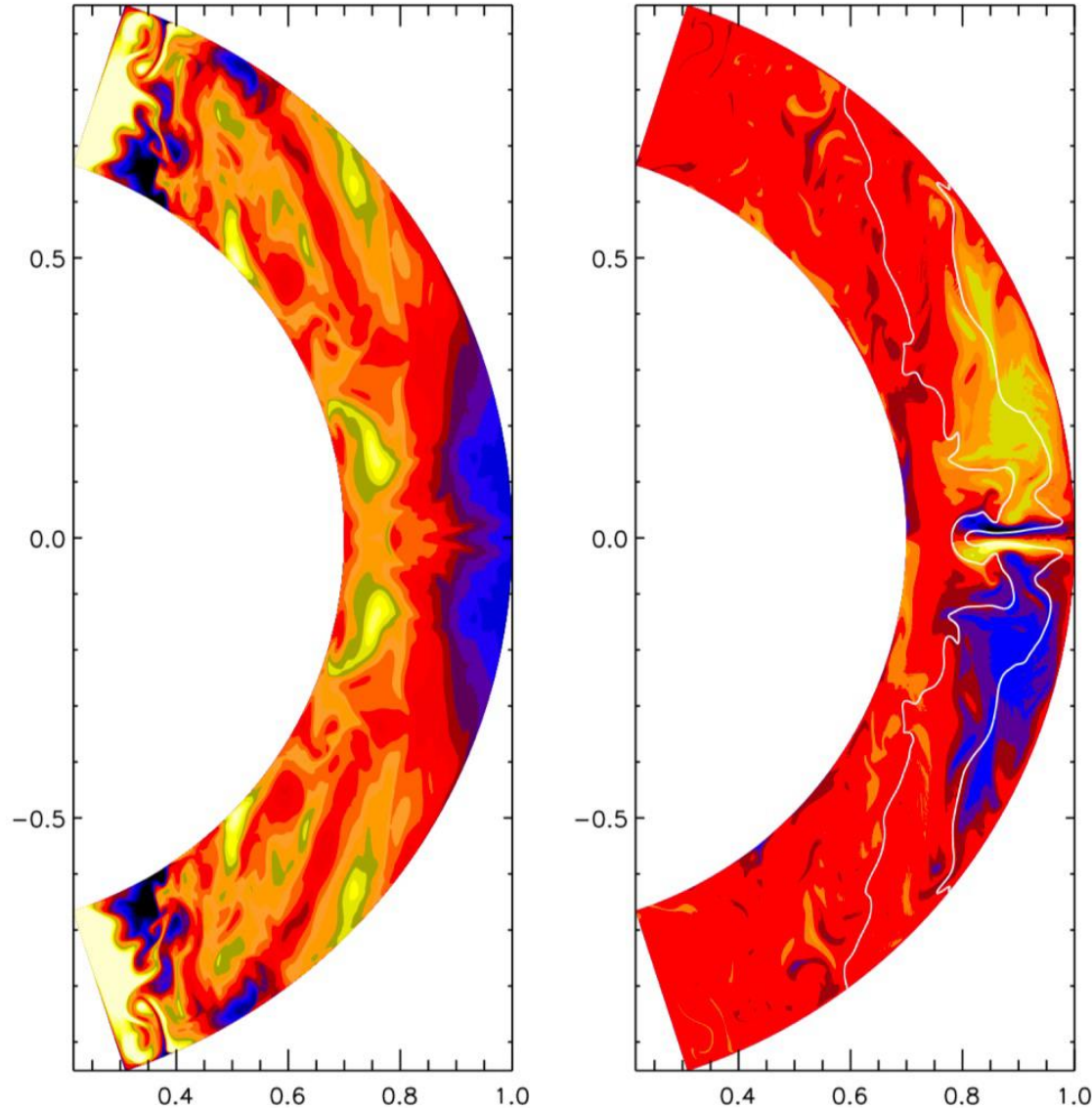


MRI in cylinders with Λ effect



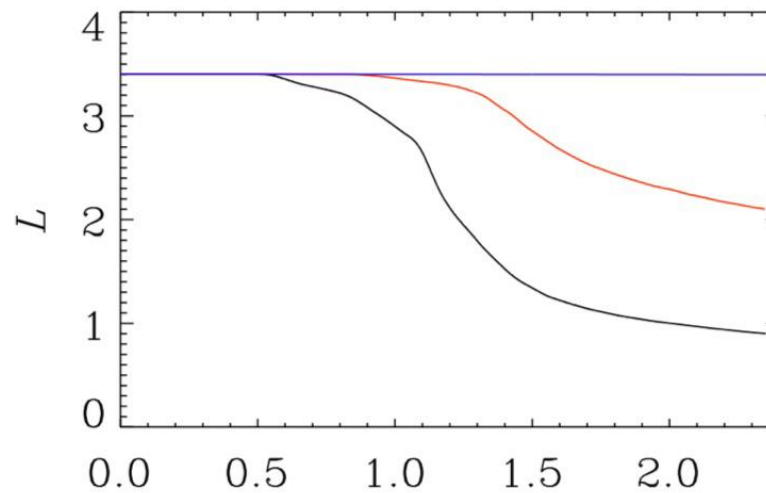
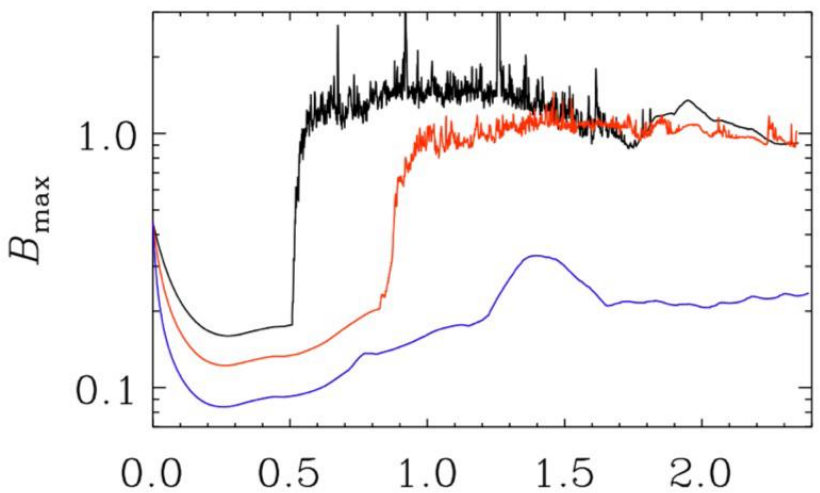
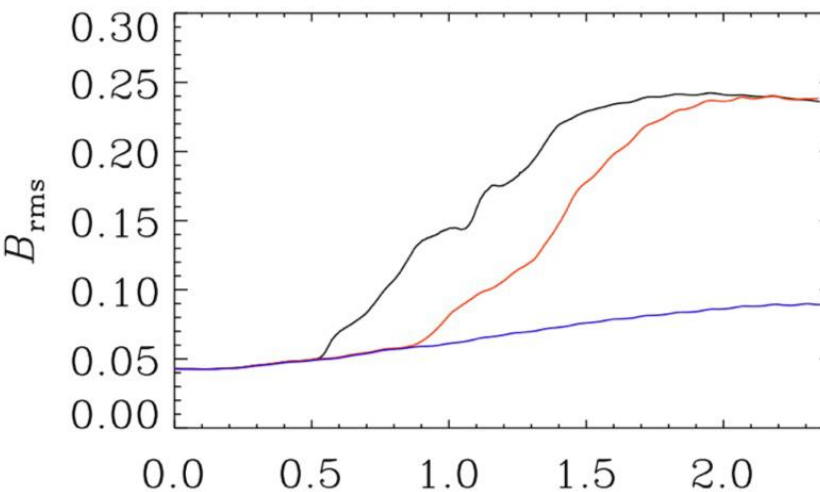
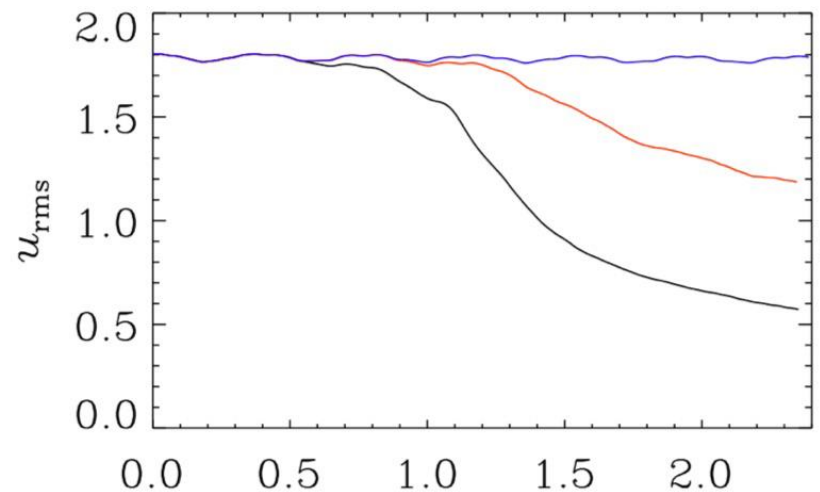
- Vertical B field
 - Reproduce vertical wavelength
- Radially extended
 - Stronger in the outer parts
- Upper row: velocity
 - Note the shear flow
- Lower row: B-field

MRI in a sphere: initial dipolar field



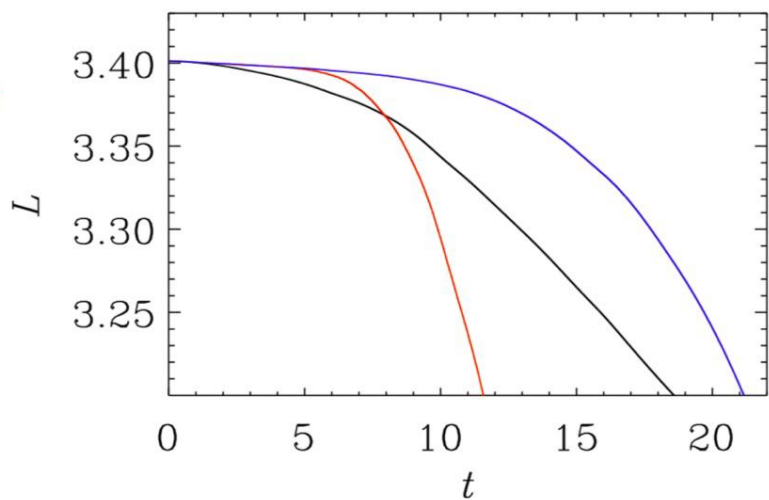
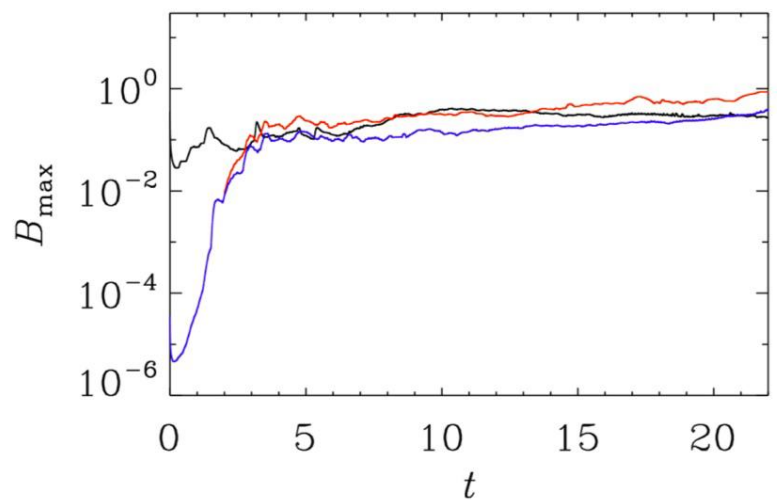
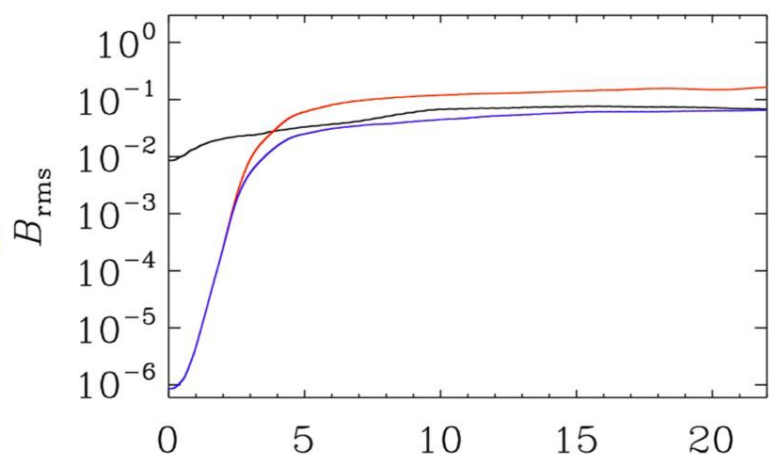
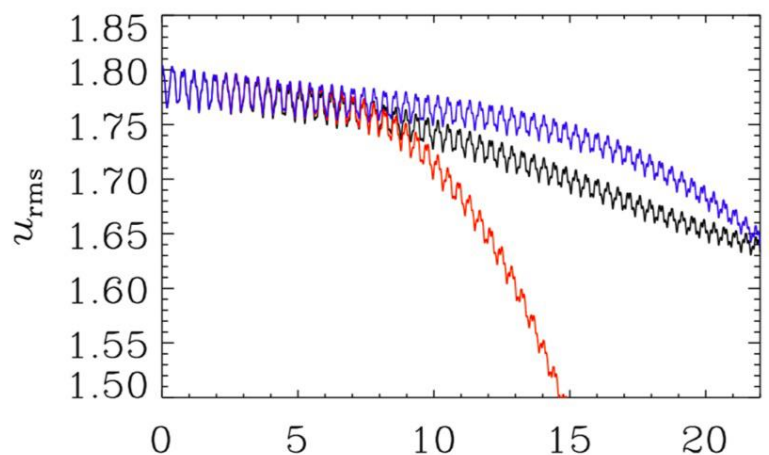
- Black: MRI
 - Reproduce vertical wavelength
- Radially extended
 - Stronger in the outer parts
- Upper row: velocity
 - Note the shear flow
- Lower row: B-field

Need huge resolution



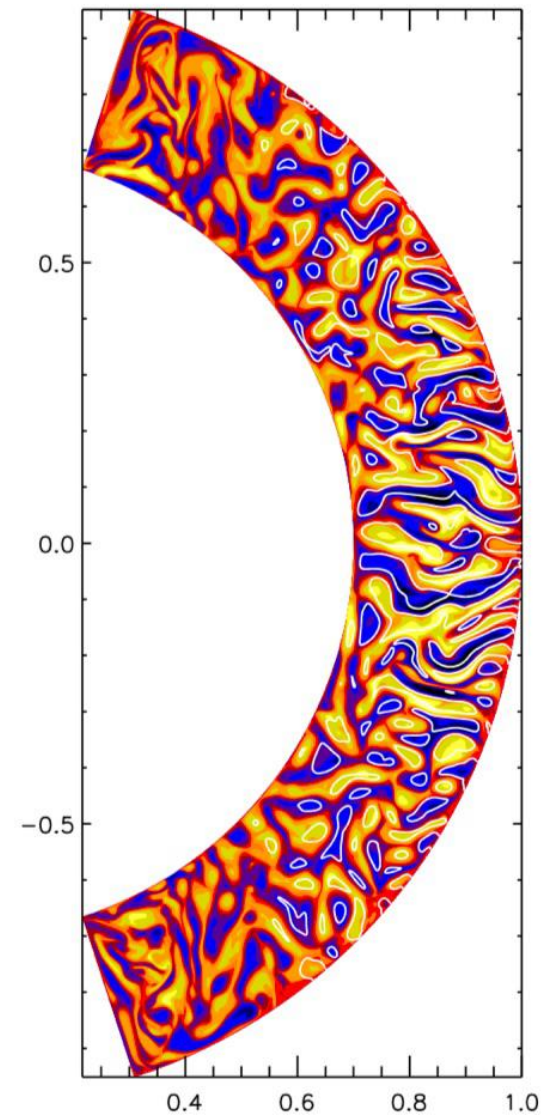
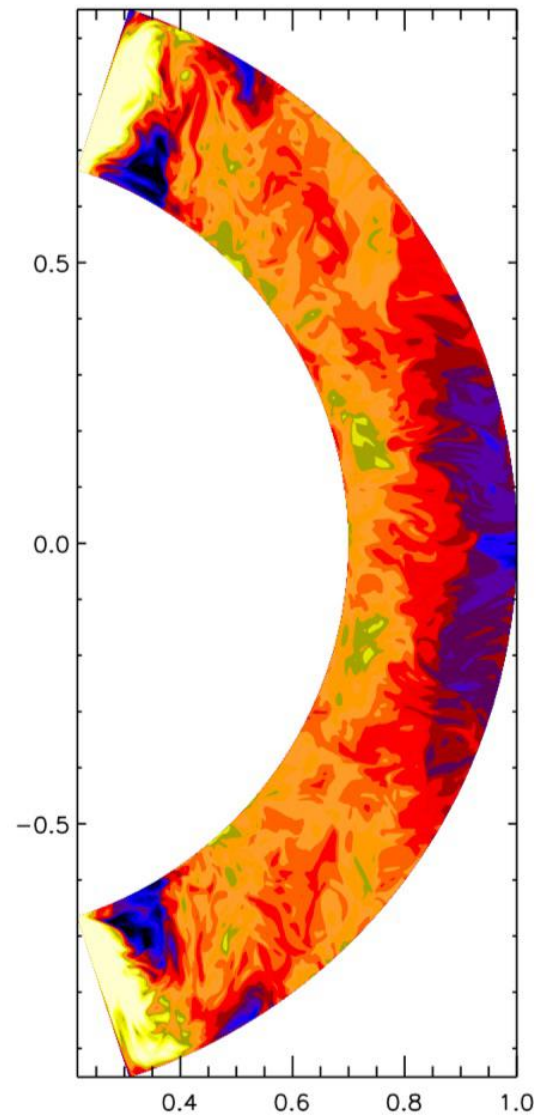
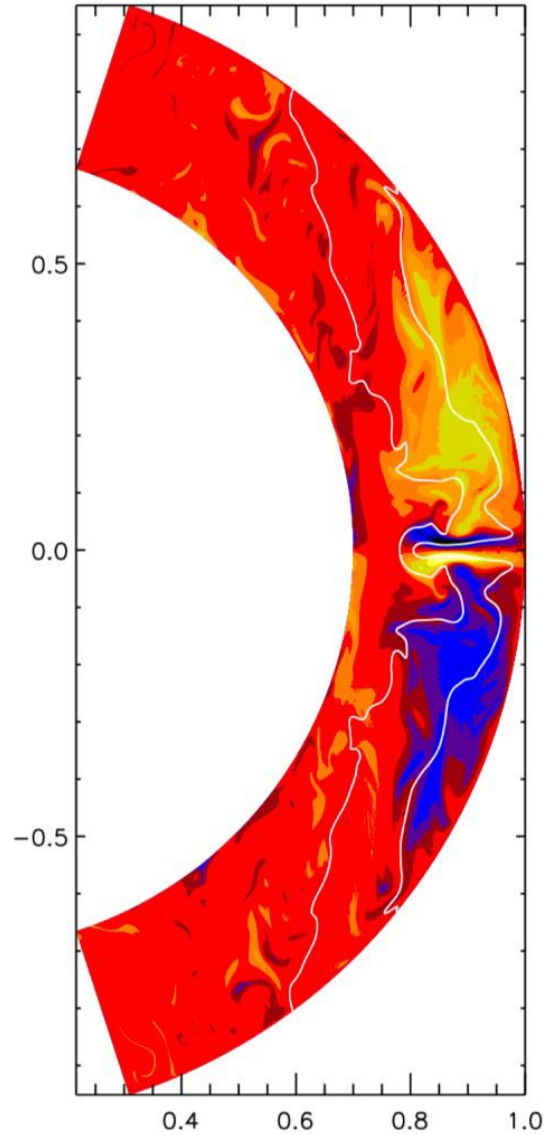
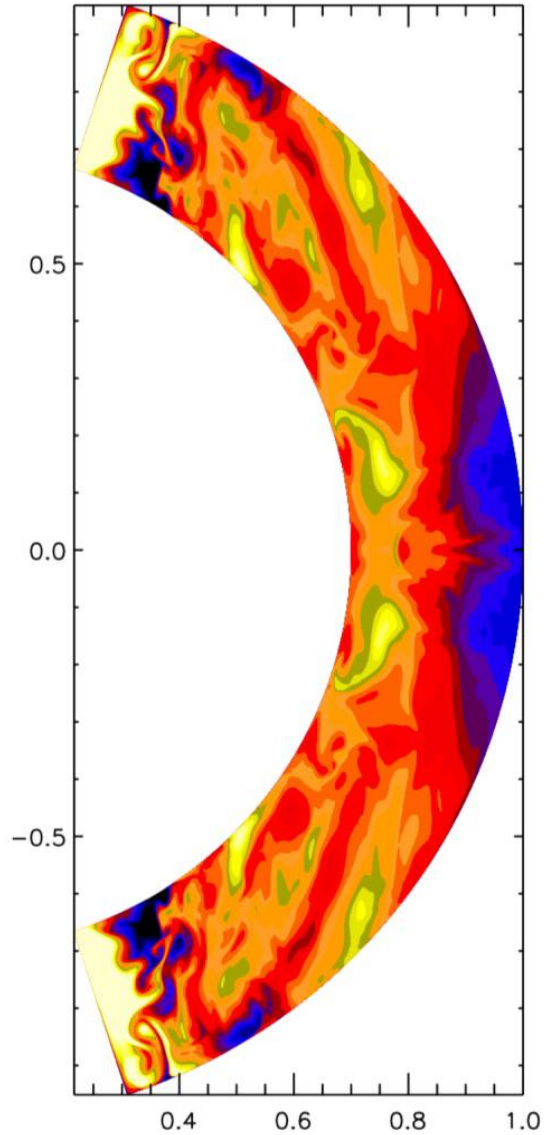
- **Black:**
 - 512x2048
 - Rapid rise
 - noise
- **Red**
 - 1024x4096
 - Still rapid rise
- **Blue**
 - 2048x8192
 - Mild increase
- Such large resolution unfamiliar in dynamos

Need to reproduce B-field with α effect dynamo



- **Black: MRI**
 - Angular momentum loss from small-scale magnetic field
- **red: dynamo run**
 - Quenching included
 - Sudden angular momentum loss
 - But consider $t < 5$
- **Blue: more quenching**
 - Angular momentum better conserved
 - Magnetic field ok
- **How does the field look like?**

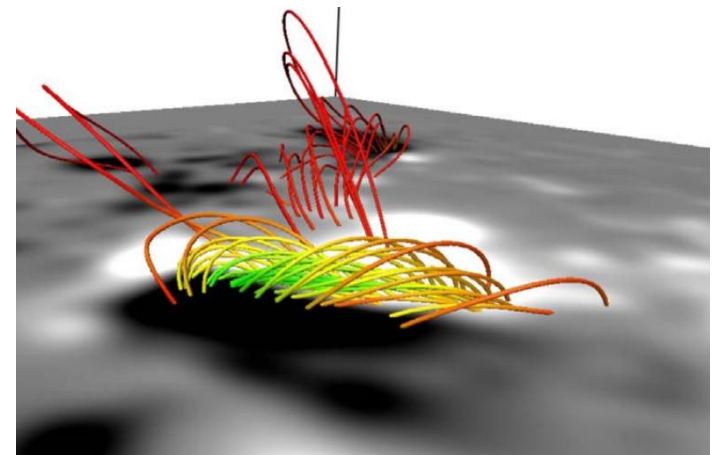
B-field from α effect much smaller scale!



Other problems with the dynamo

What	How	But?
Equatorward migration?	Near-surface shear layer	But dynamo from deeper layers usually stronger
	Meridional circulation	Pattern in simulations? Stellar cycle period?
Magnetic helicity ejection?	Coronal mass ejections	Also loss of large-scale field

- Emergent twist seems very strong
 - Where from?
- Theoretically expected from dynamos
 - But maybe not so much
 - Alleviates catastrophic quenching

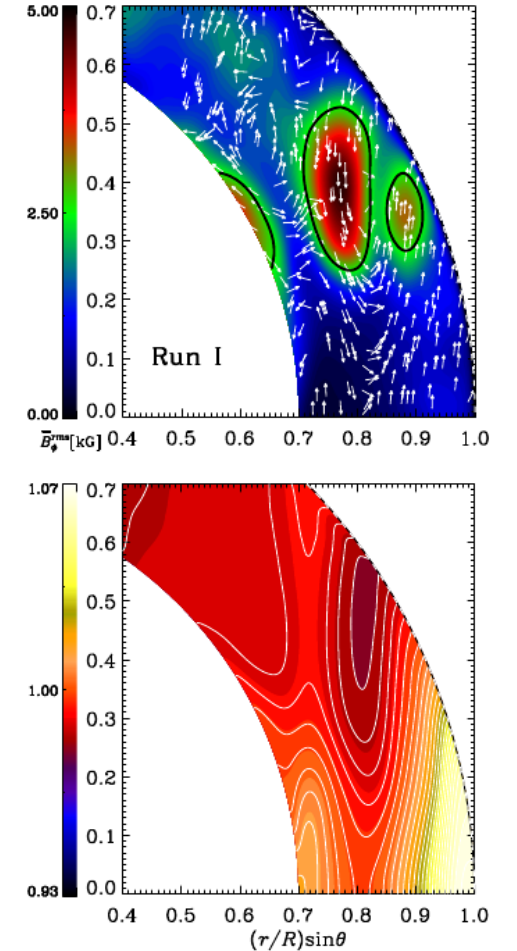
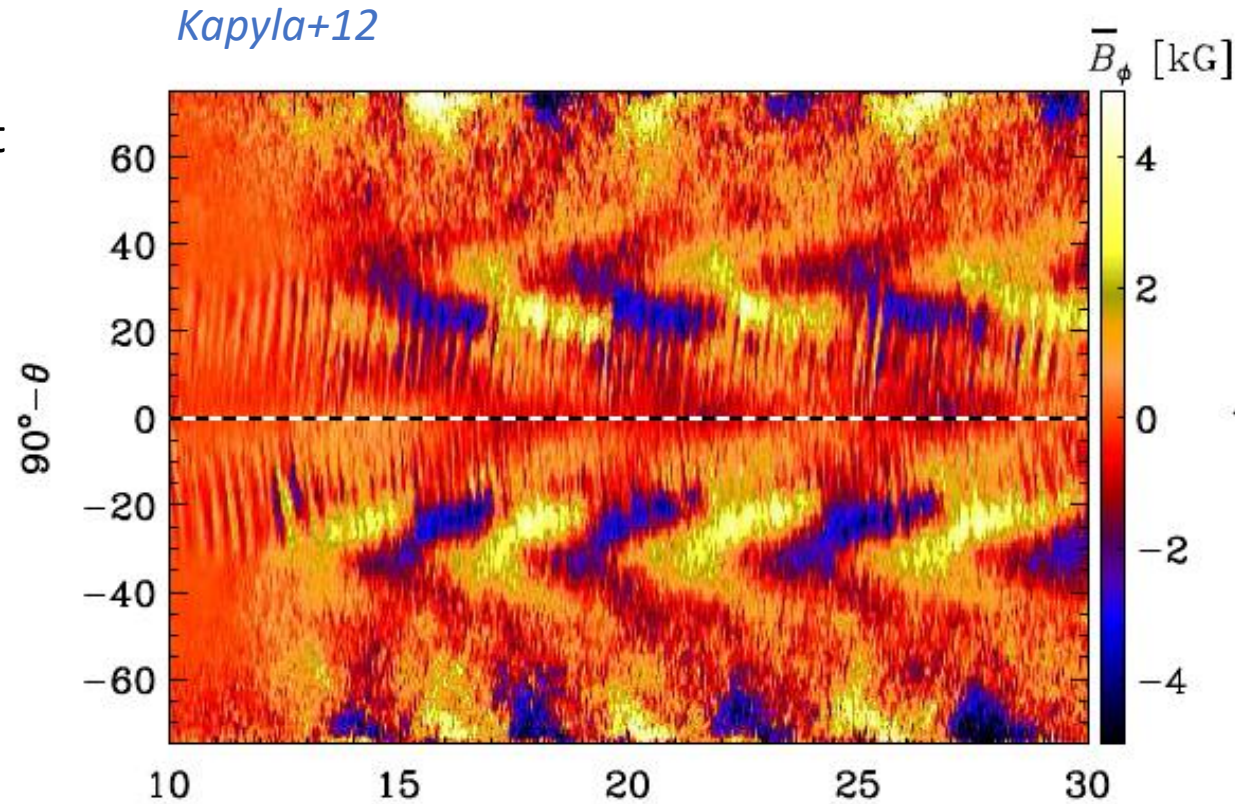


Vemareddy 2019 (NLFFF for AR12673)

Equatorward migration

Warnecke+14

- Historically most important
 - Requires $\alpha > 0$, and
 - $d\Omega/dr < 0$
 - But: helioseismology
- Alternatives
 - Overshoot dynamo
 - Surface shear layer
 - Meridional circulation



- Caused here by negative radial shear at mid-latitudes
 - Exactly as predicted by Parker-Yoshimura: $\alpha d\Omega/dr < 0$
 - Not seen in helioseismology
 - Meridional circulation: multiple cylindrical shells (also not seen in seismology)

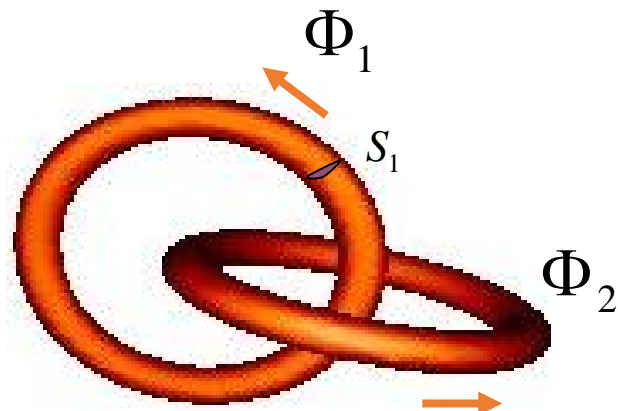
Magnetic helicity conservation

$$\frac{d}{dt} \left\langle \frac{1}{2} \mathbf{B}^2 \right\rangle = - \left\langle \mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) \right\rangle - \eta \left\langle \mathbf{J}^2 \right\rangle$$

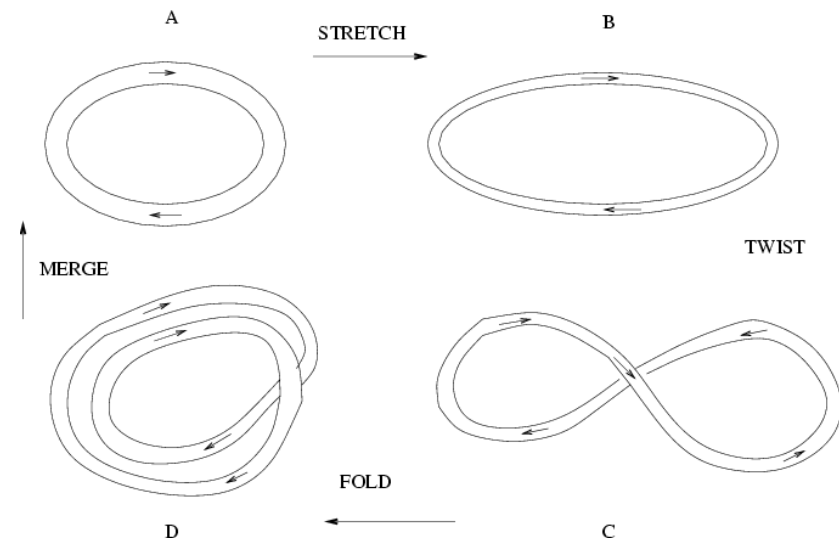
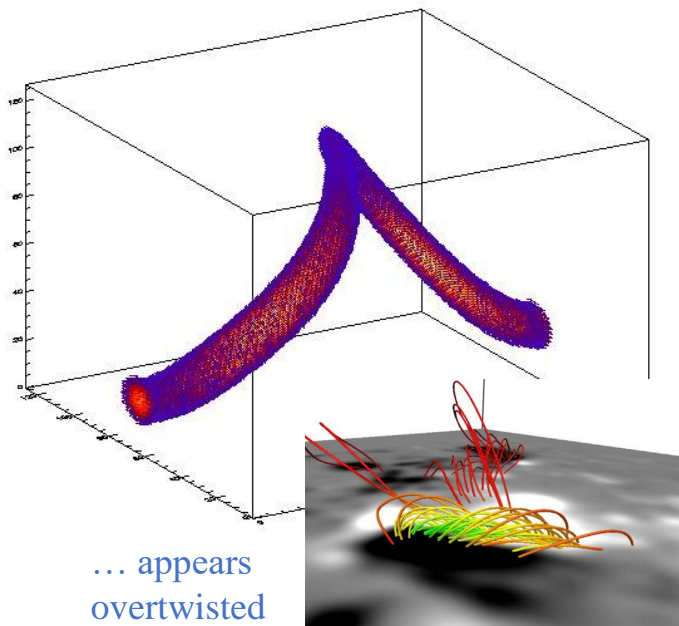
J diverges as $\eta \rightarrow 0$

$$J \propto \eta^{-1/2} \propto kB \propto k$$

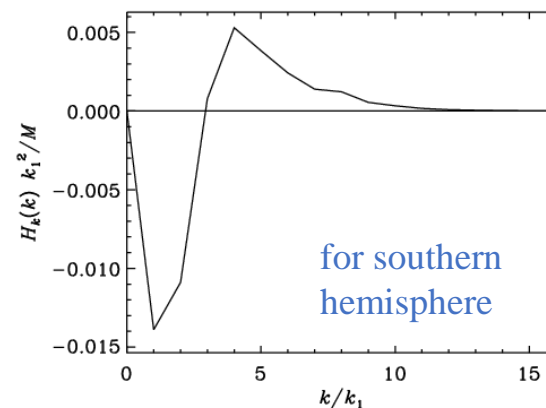
$$\frac{1}{2} \frac{d}{dt} \langle \mathbf{A} \cdot \mathbf{B} \rangle = - \left\langle \mathbf{u} \cdot (\mathbf{B} \times \mathbf{B}) \right\rangle - \eta \langle \mathbf{J} \cdot \mathbf{B} \rangle \rightarrow \eta \eta^{-1/2} = \eta^{1/2} \rightarrow 0$$



$$H = \pm 2\Phi_1\Phi_2$$



$$\int H(k) dk = \langle \mathbf{A} \cdot \mathbf{B} \rangle = \langle \bar{\mathbf{A}} \cdot \bar{\mathbf{B}} \rangle + \langle \mathbf{a} \cdot \mathbf{b} \rangle$$



Expell as waste



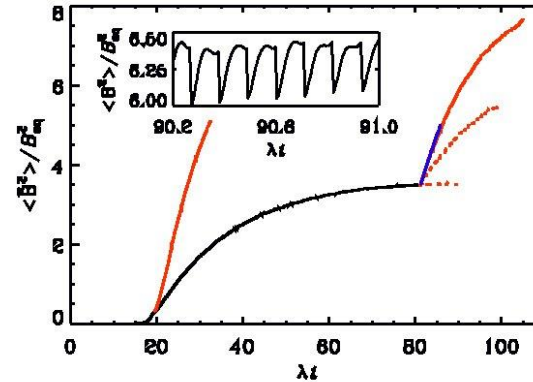
for southern hemisphere

Catastrophic quenching with/without fluxes

$$\frac{d}{dt} \langle \mathbf{A} \cdot \mathbf{B} \rangle = -2\eta \langle \mathbf{J} \cdot \mathbf{B} \rangle$$

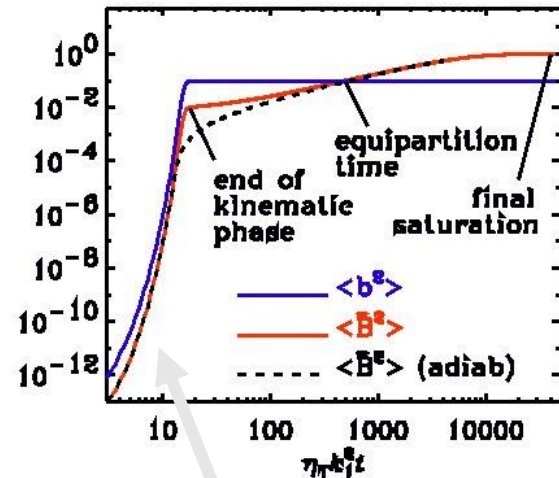
$$k_m^{-1} \frac{d}{dt} \langle \overline{\mathbf{B}^2} \rangle = -2\eta_m k_m \langle \overline{\mathbf{B}^2} \rangle + 2\eta_f k_f \langle \mathbf{b}^2 \rangle$$

$$\langle \overline{\mathbf{B}^2} \rangle = \langle \mathbf{b}^2 \rangle \frac{\eta_f k_f}{\eta_m k_m} \left[1 - e^{-2\eta_m k_m^2 (t-t_s)} \right]$$



Numerical experiment:
remove field for $k > 4$
every 1-3 turnover times

- 1) higher saturation level
- 2) still slow time scale



$$\langle \overline{\mathbf{J}} \cdot \overline{\mathbf{B}} \rangle + \langle \mathbf{j} \cdot \mathbf{b} \rangle = 0$$

Significant field already after kinematic growth phase

followed by slow resistive adjustment

$$\langle \overline{\mathbf{A}} \cdot \overline{\mathbf{B}} \rangle + \langle \mathbf{a} \cdot \mathbf{b} \rangle = 0$$

$$\frac{\partial \overline{h}_m}{\partial t} = 2\overline{\mathcal{E}} \cdot \overline{\mathbf{B}} - 2\eta \overline{\mathbf{J}} \cdot \overline{\mathbf{B}} - \nabla \cdot (\overline{\mathcal{F}}'_m - \overline{\mathcal{E}} \times \overline{\mathbf{A}}),$$

$$\frac{\partial \overline{h}_f}{\partial t} = -2\overline{\mathcal{E}} \cdot \overline{\mathbf{B}} - 2\eta \overline{\mathbf{j}} \cdot \overline{\mathbf{b}} - \nabla \cdot (\overline{\mathcal{F}} - \overline{\mathcal{F}}'_m + \overline{\mathcal{E}} \times \overline{\mathbf{A}}),$$

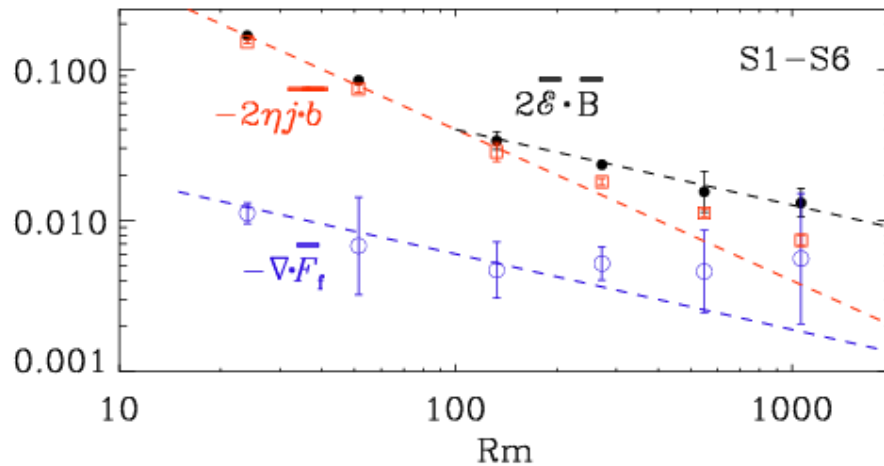
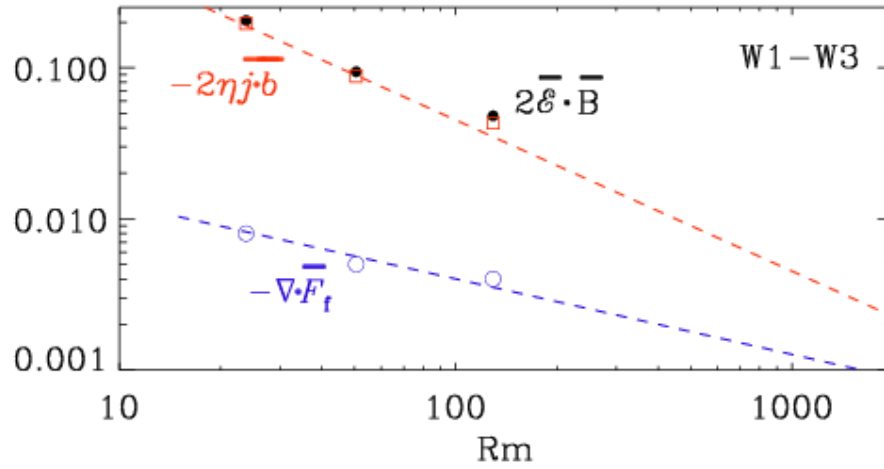
Magnetic helicity flux in simulations

$$\frac{d}{dt} \langle \bar{\mathbf{A}} \cdot \bar{\mathbf{B}} \rangle = +2 \langle \bar{\boldsymbol{\varepsilon}} \cdot \bar{\mathbf{B}} \rangle - 2\eta \langle \bar{\mathbf{J}} \cdot \bar{\mathbf{B}} \rangle - \nabla \cdot \boldsymbol{\mathcal{F}}_m$$

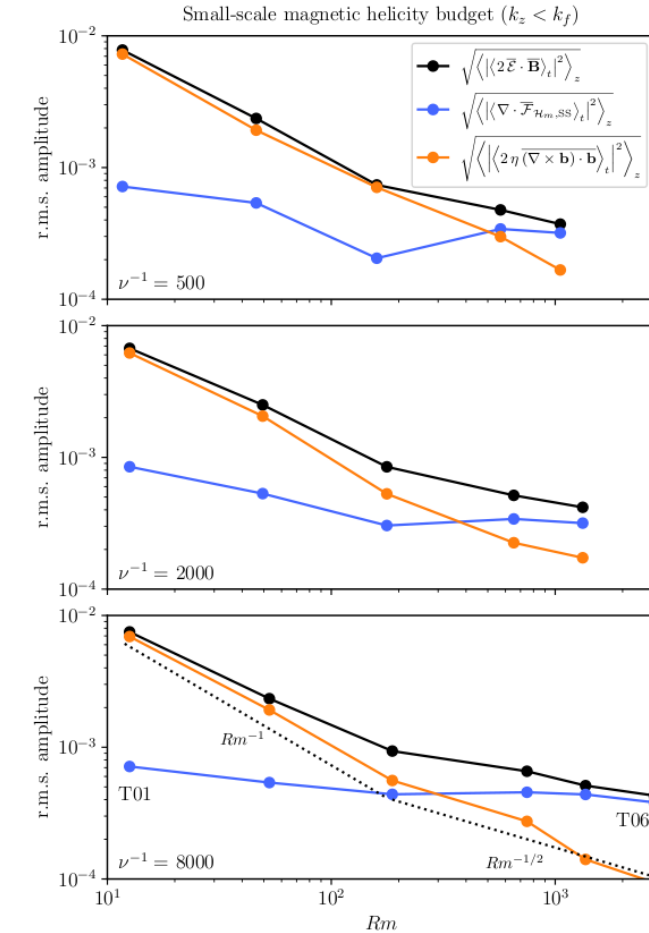
$$\frac{d}{dt} \langle \mathbf{a} \cdot \mathbf{b} \rangle = -2 \langle \bar{\boldsymbol{\varepsilon}} \cdot \bar{\mathbf{B}} \rangle - 2\eta \langle \mathbf{j} \cdot \mathbf{b} \rangle - \nabla \cdot \boldsymbol{\mathcal{F}}_f$$

- EMF and resistive terms still dominant
 - Fluxes import at large $Rm \sim 1000$
 - Rm based on k_f
 - Smaller by 2π
- Here for galactic wind
 - Still Rm dependence
 - Cannot be ideal

Gauge-invariant in steady state!

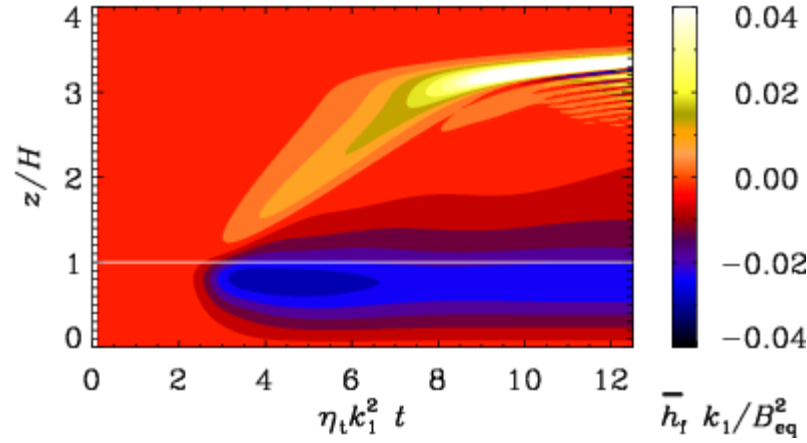
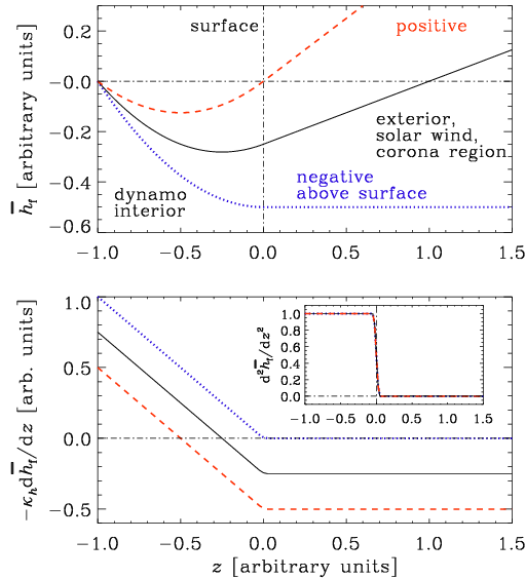


Rincon21: equatorial fluxes



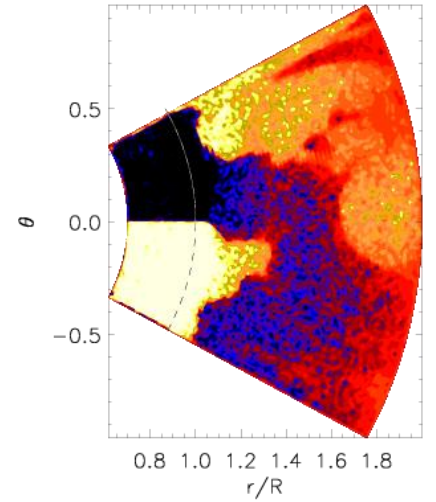
To carry negative flux: need positive gradient

$$\bar{\mathbf{E}}_f = -\kappa_h \nabla \bar{h}_f$$



Brandenburg, Candelaresi, Chatterjee
(2009, MNRAS 398, 1414)

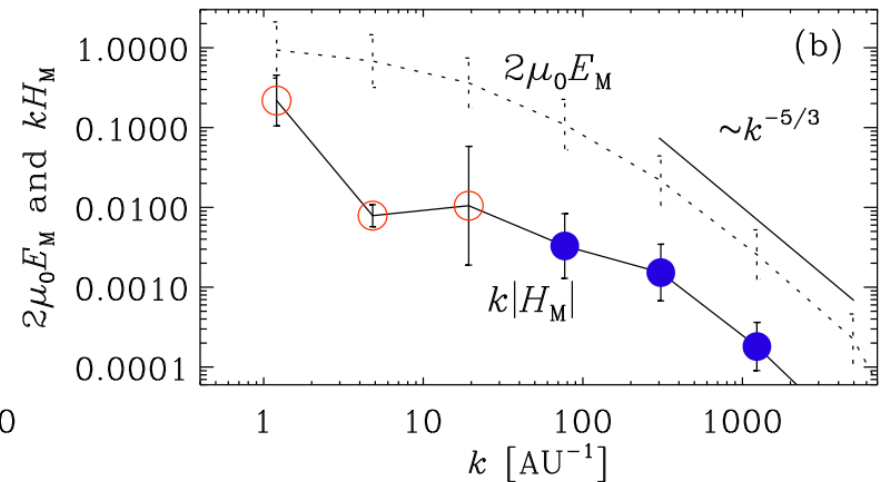
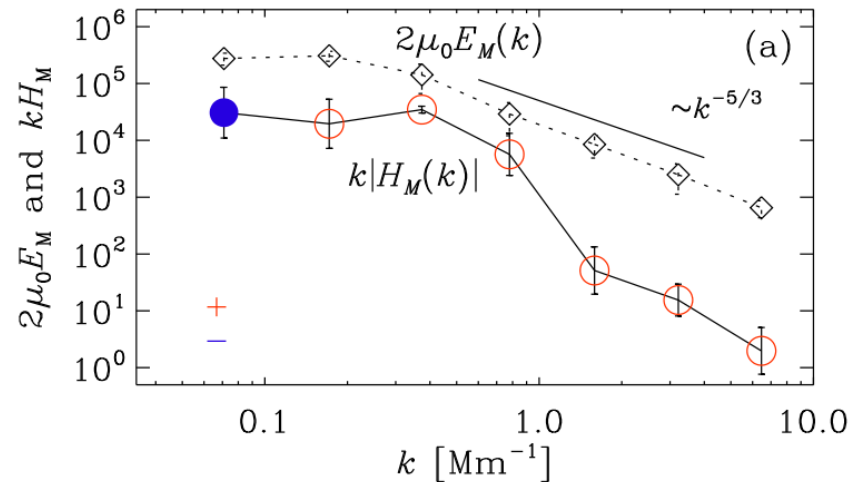
- Reversals of magnetic helicity above dynamo
- Surprising consequence of turbulent helicity fluxes



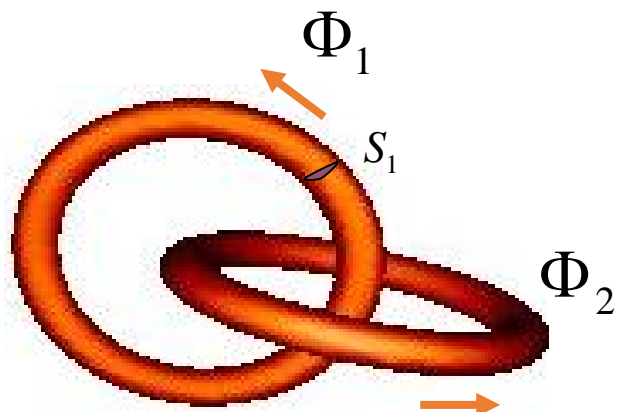
$$H_M^{1D}(k_R) = 4 \text{Im}(\hat{B}_T \hat{B}_N^*) / k_R.$$

Matthaeus+Goldstein+Smith82

Brandenburg+11 (ApJ)
Zhang+14 (ApJ)



Magnetic helicity



$$H = \int_V \mathbf{A} \cdot \mathbf{B} dV$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

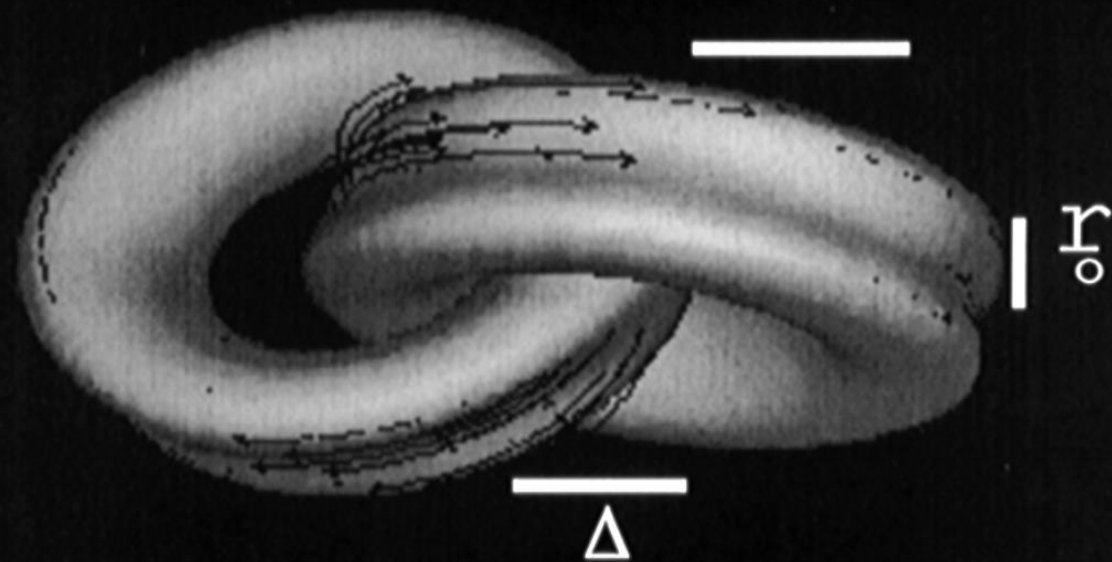
$$H = \pm 2\Phi_1\Phi_2$$

*Therefore the unit is
Maxwell squared*

$$H_1 = \int_{L_1} \mathbf{A} \cdot d\ell \int_{S_1} \mathbf{B} \cdot d\mathbf{S}$$

$$= \int_{S_2} \nabla \times \mathbf{A} \cdot d\mathbf{S} = \Phi_2 \quad = \Phi_1$$

t=2



t=3



Nonhelical decay: mag helicity in patches conserved

$h(x) = \mathbf{A} \cdot \mathbf{B}$

$$\mathcal{I}_H(R \ll \xi_M) \simeq \int_{V_n} d^3r \langle h(\mathbf{x}) h(\mathbf{x}) \rangle \propto R^3$$

Hosking
integral

$$\mathcal{I}_H(R) = \int_0^\infty dk w_{\text{sph}}^{\text{BC}}(k) \text{Sp}(h)$$

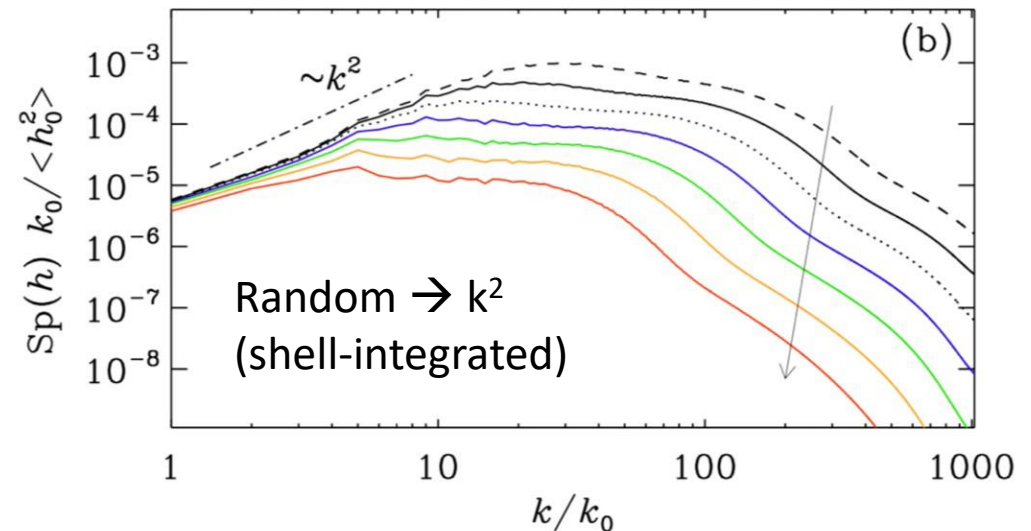
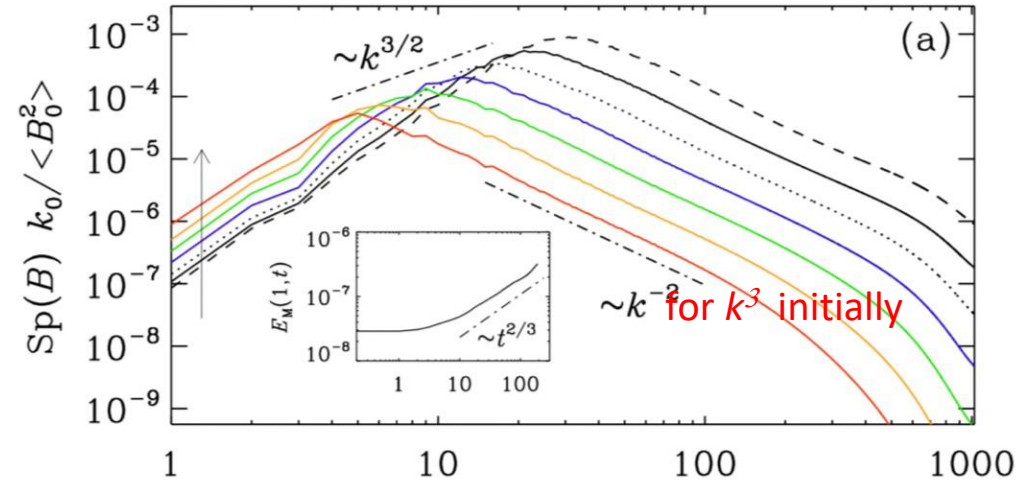
$$\text{Sp}(h) = \frac{1}{V} \frac{k^2}{(2\pi)^3} \int_{|k|=k} d\Omega_k \tilde{h}^*(\mathbf{k}) \tilde{h}(\mathbf{k})$$

$$\text{Sp}(h) = \frac{I_H}{2\pi^2} k^2 + \mathcal{O}(k^4)$$

$$[I_H] = \text{cm}^9 \text{s}^{-4}$$

$$\xi_M = I_H^a t^b$$

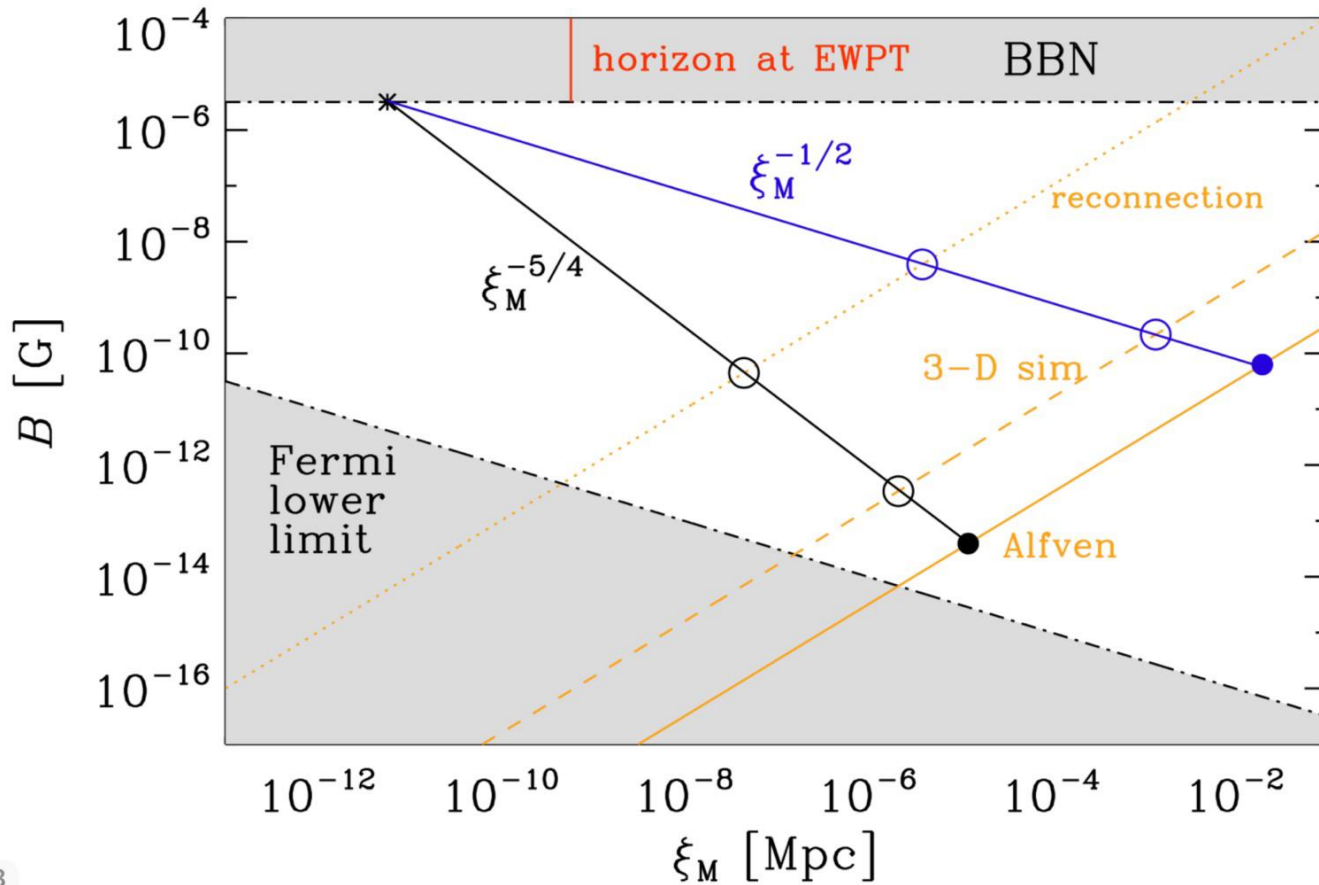
$$a=1/9, b=4/9$$



AB, Sharma, Vachaspati (2023)

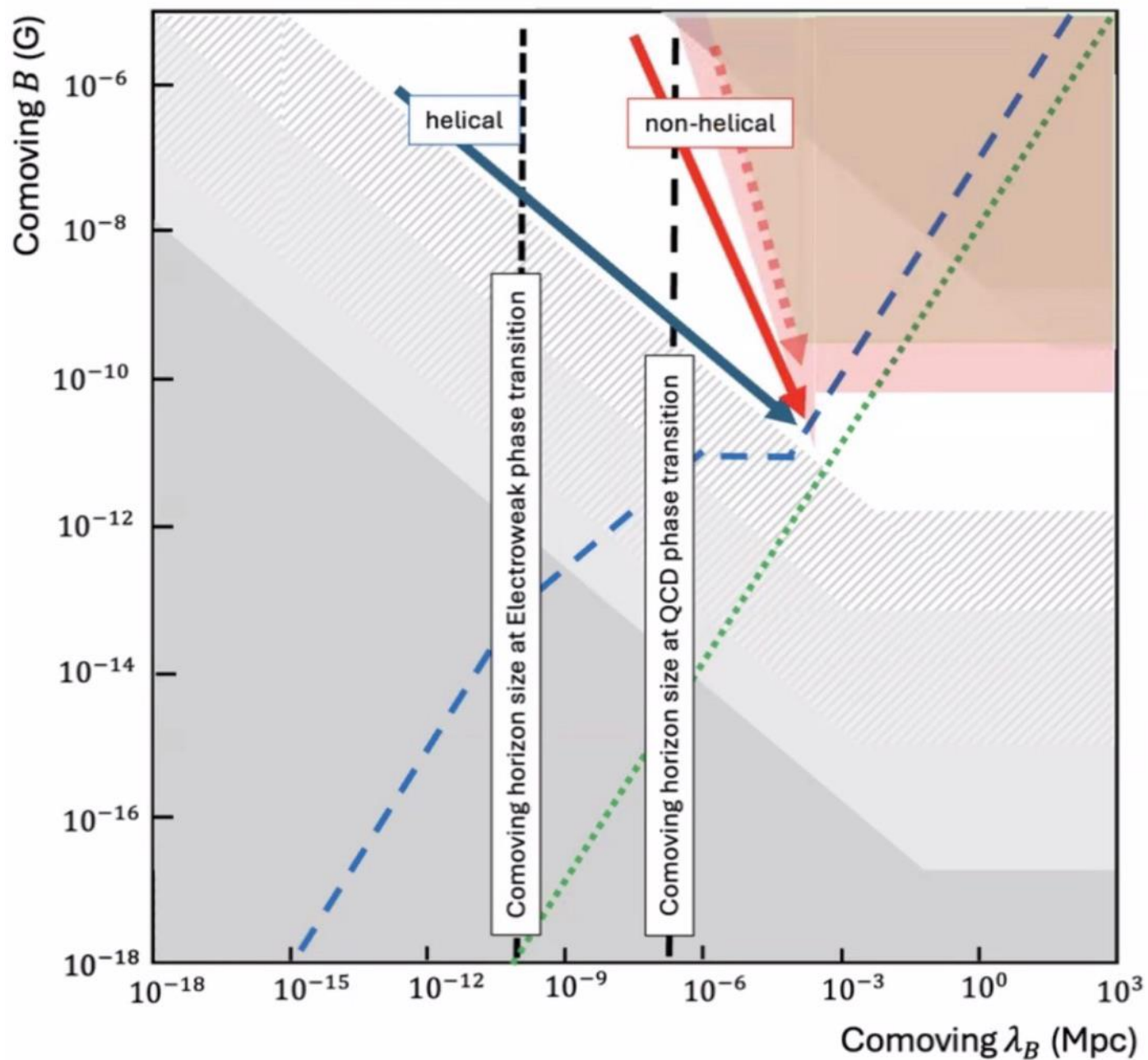
$$\xi_M(t) \approx 0.12 I_H^{1/9} t^{4/9}, \quad \mathcal{E}_M(t) \approx 3.7 I_H^{2/9} t^{-10/9}, \quad E_M(k, t) \lesssim 0.025 I_H^{1/2} (k/k_0)^{3/2}$$

Resistively prolonged decay during radiative era



- Endpoints under assumption that decay time = Alfven time
- Use: decay time = recombination time
- Possibility: decay time \gg Alfven time
- \rightarrow Premature endpoint of evolution

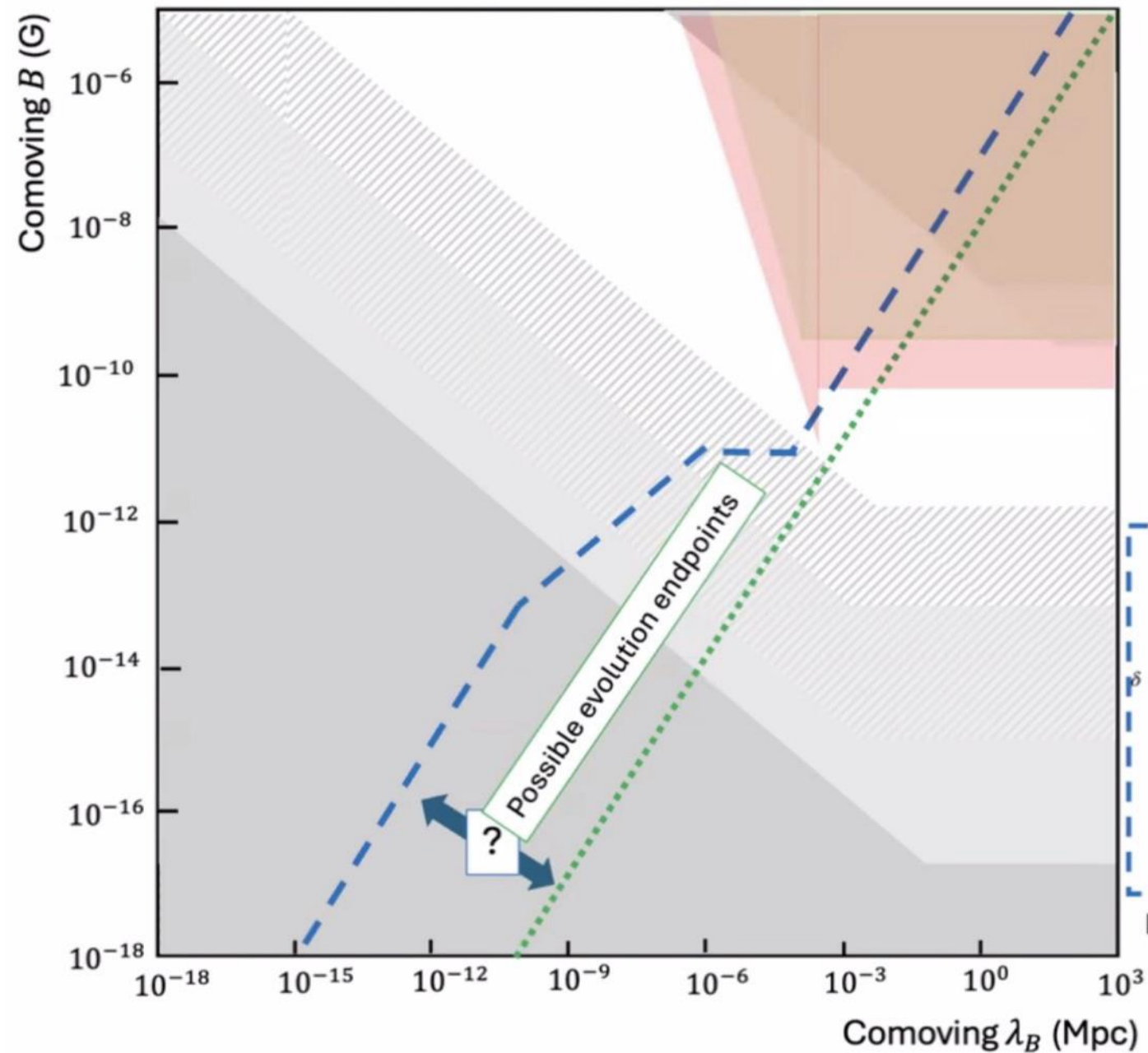
Magnetic field at the moment of generation



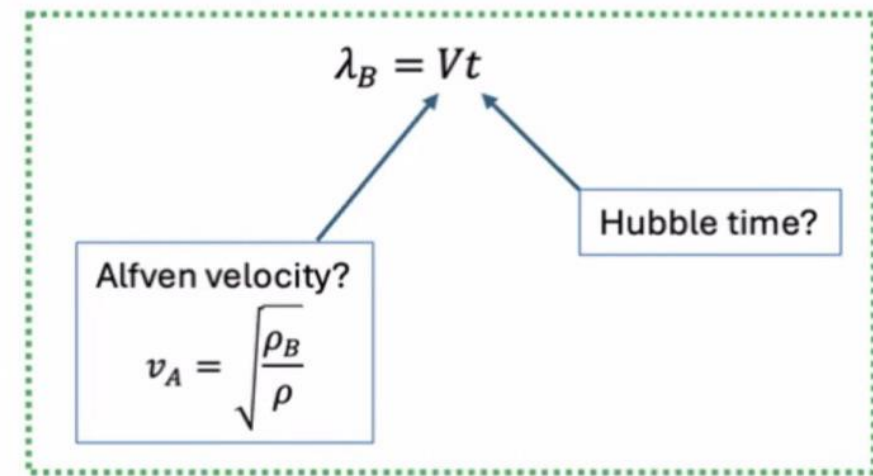
Backtracing the magnetic field trajectory we can guess from which epoch does magnetic field originate.

Example: non-helical magnetic field consistent with the CMB / Hubble tension hint has to originate from the QCD epoch.

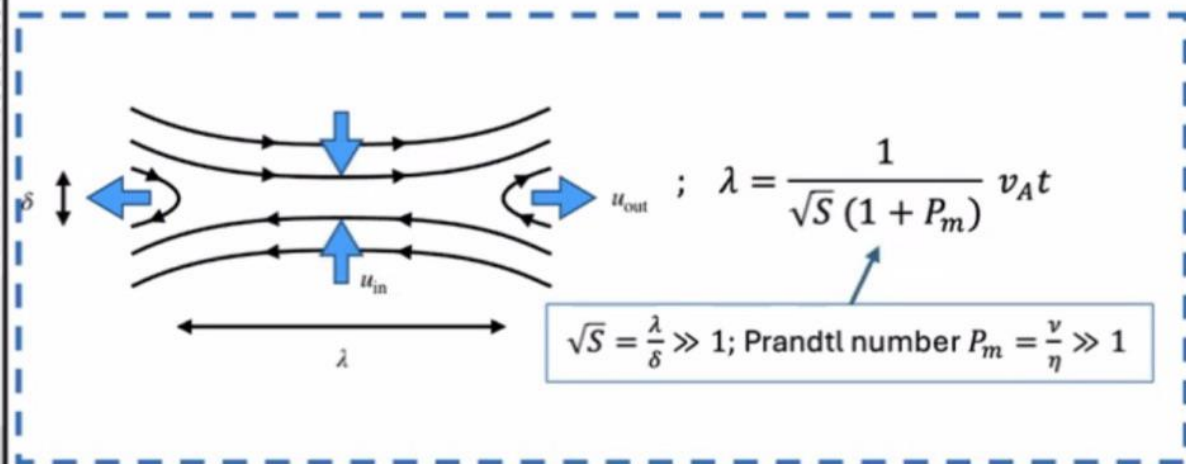
Backtracing of magnetic field evolution



“Largest processed turbulent eddy” concept:



Banerjee, Jedamzik, astro-ph/0410032



Hosking, Schekochihin, 2203.03573

Resistively controlled primordial magnetic turbulence decay

A. Brandenburg^{1,2,3,4,5}, A. Neronov^{6,7}, and F. Vazza^{8,9,10}

Relation between decay time

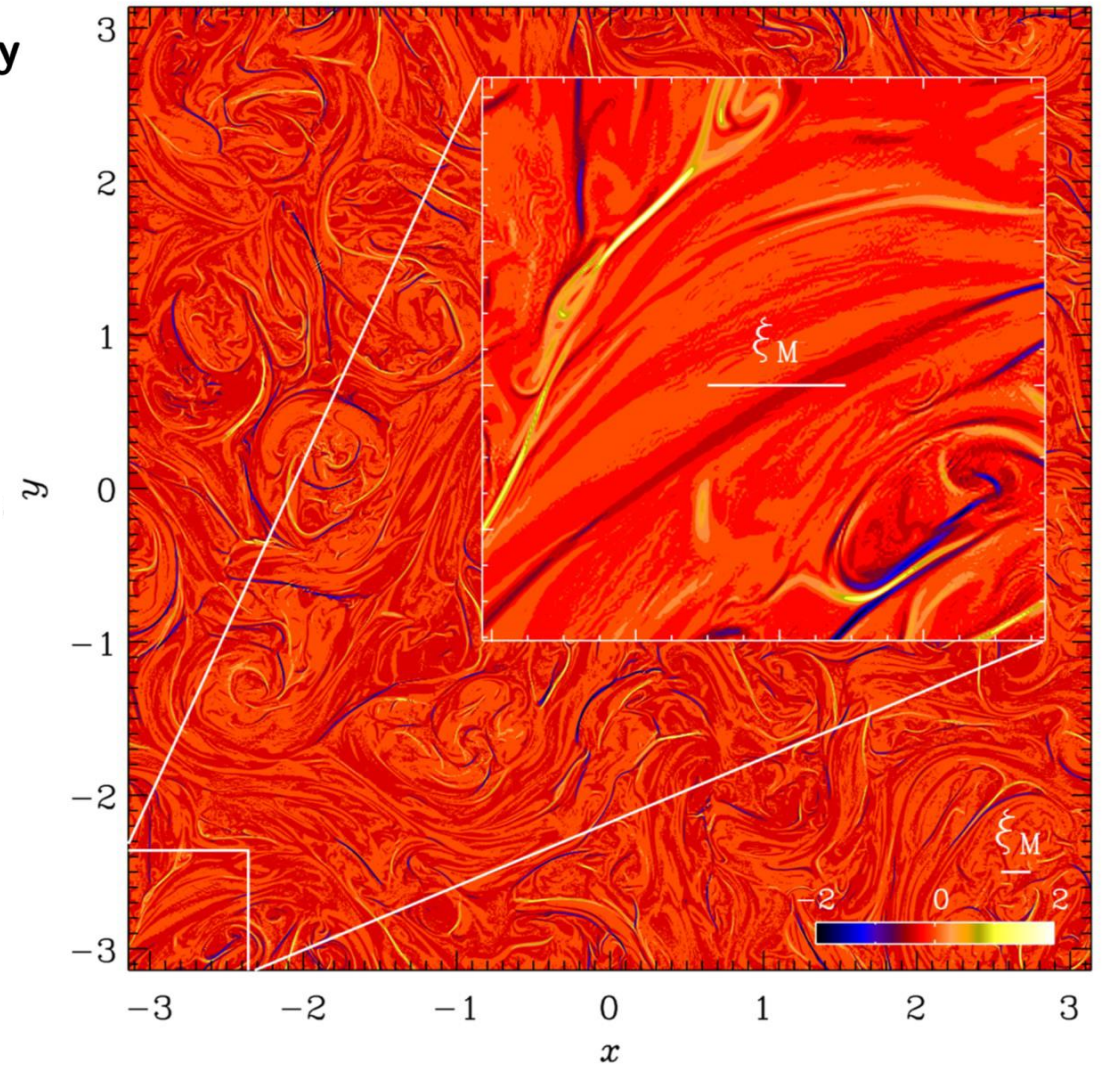
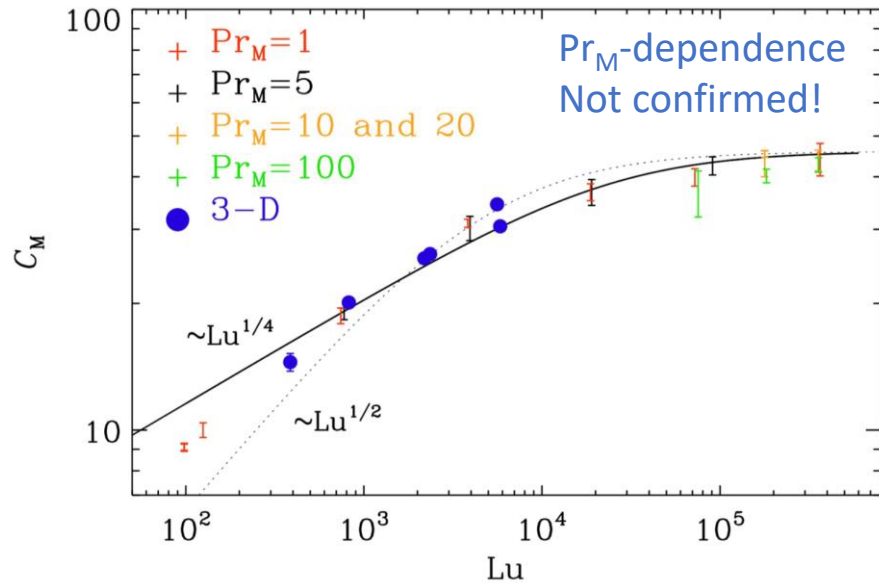
$$\tau^{-1} = -d \ln \mathcal{E}_M / dt$$

and Alfvén time

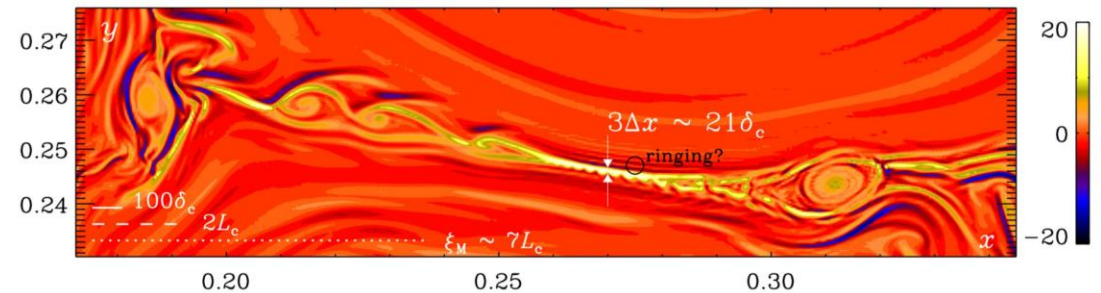
$$\tau_A = \xi_M / v_A \quad \mathcal{E}_M = B_{\text{rms}}^2 / 2\mu_0 = \rho v_A^2 / 2$$

Determine C_M in relation:

$$\tau = C_M \xi_M / v_A$$

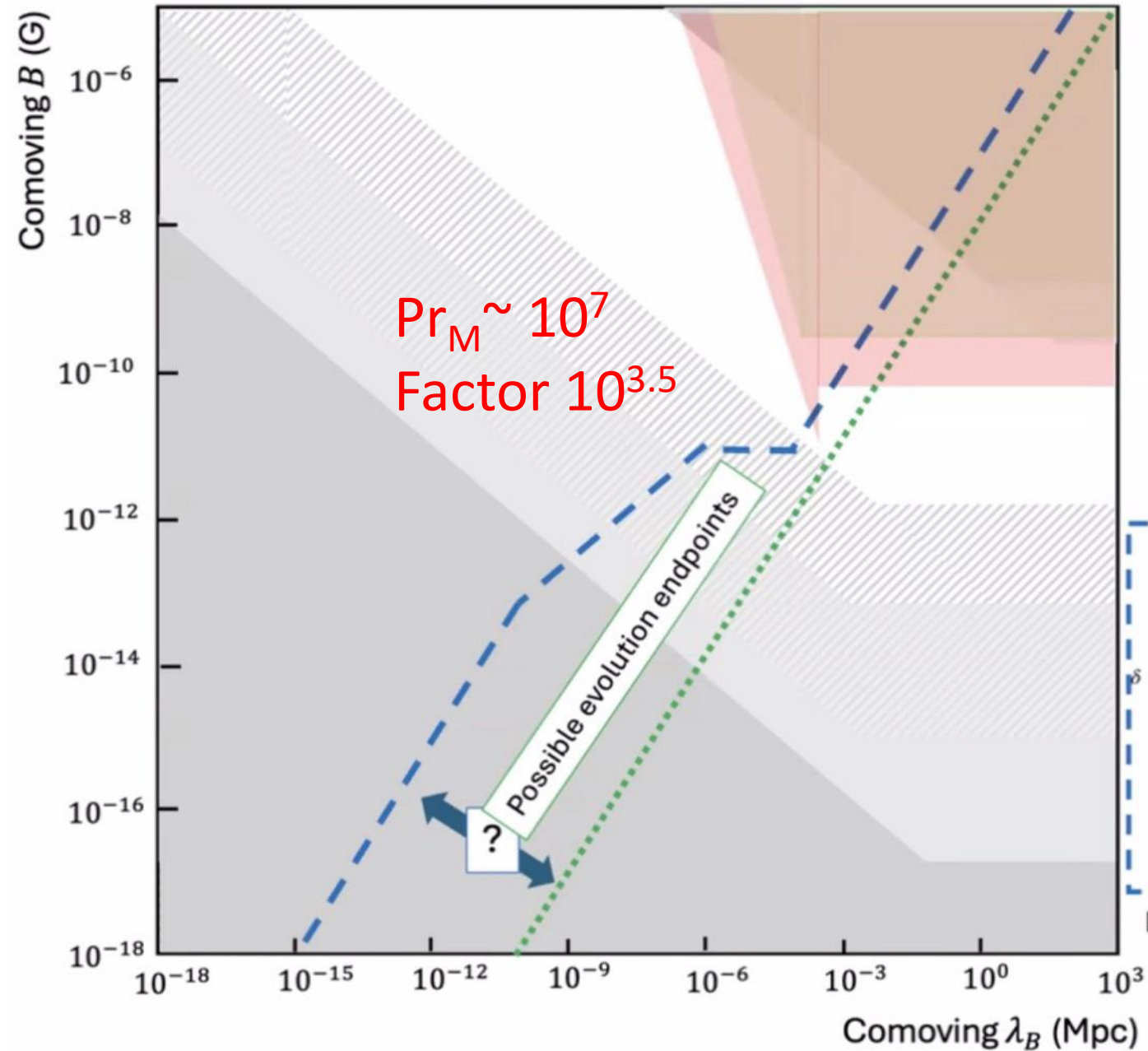


3-D

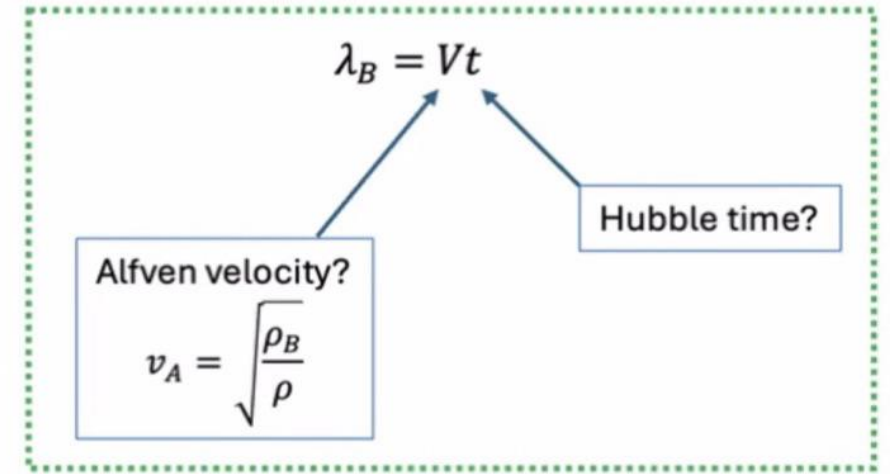


2-D

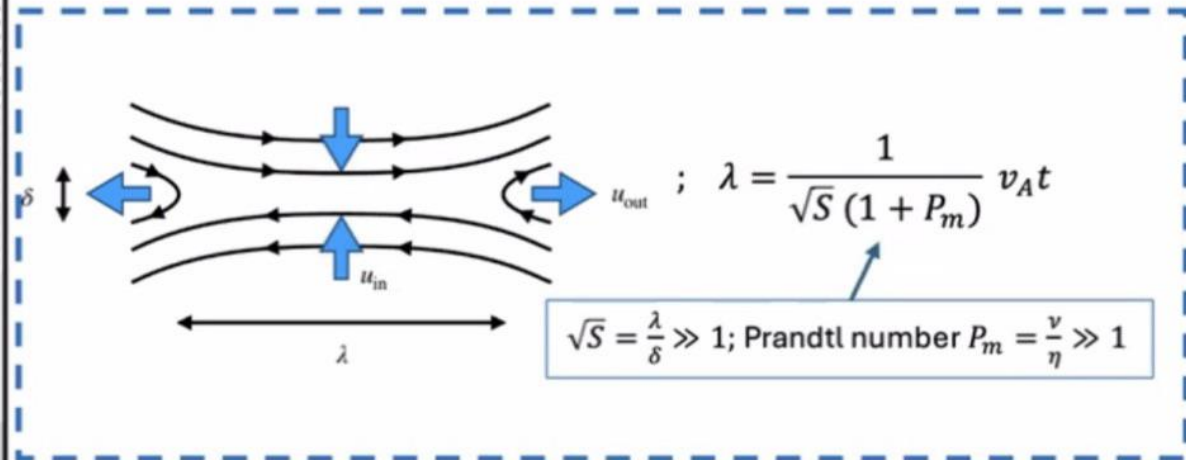
Backtracing of magnetic field evolution



“Largest processed turbulent eddy” concept:



Banerjee, Jedamzik, astro-ph/0410032



Hosking, Schekochihin, 2203.03573

Structures highly dynamical:
outflow not opposed by viscosity

Final thoughts

- Should eventually reproduce the Sun (& stars!)
- Equatorward migration: mechanism?
- Do young stars really show 2 cycles?
- Are there 2 dynamo locations?
- What exactly is wrong with simulations?
 - Are cylindrical contours important
 - How important is NSSL?
 - Can spots be made in situ? – not today
 - Subadiabatic convection important?
- Helicity is present: more than expected, different sign
 - Relation to Babcock-Leighton?
 - Does it slow down MHD turbulence & large-scale dynamos?
 - Does it stop above large Lundquist numbers $\sim 10^4$???